Prudent Price-Responsive Demands

------- ACM SIGEnergy Graduate Seminar

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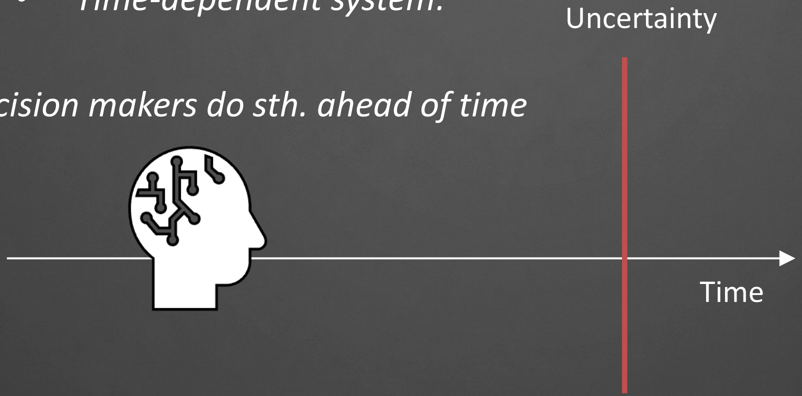
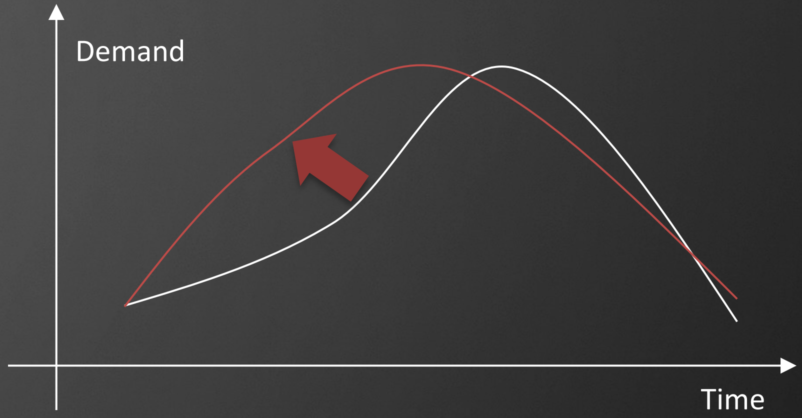
Prudent Price-Responsive Demands

- *Economic: seeing ahead, sagacity*
- *If uncertain events happen, prudent decision-makers will do sth. to respond to the event*
- *Time-dependent system:*

Decision makers do sth. ahead of time



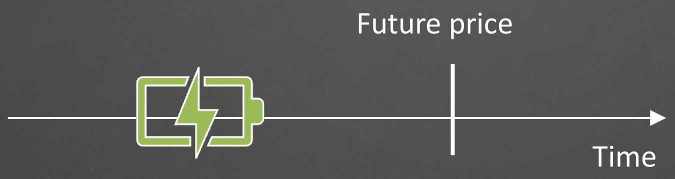


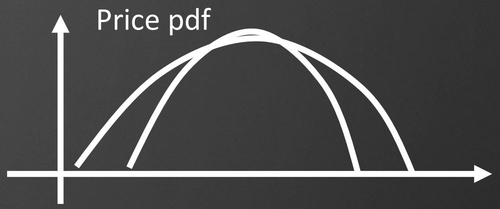
Toy example:

Suppose you have a battery and participate in the real-time market with price uncertainty.

You need to decide on charge or discharge starting now until the price is realized

Now, I told you future price variance increase, but the expectation is the same.





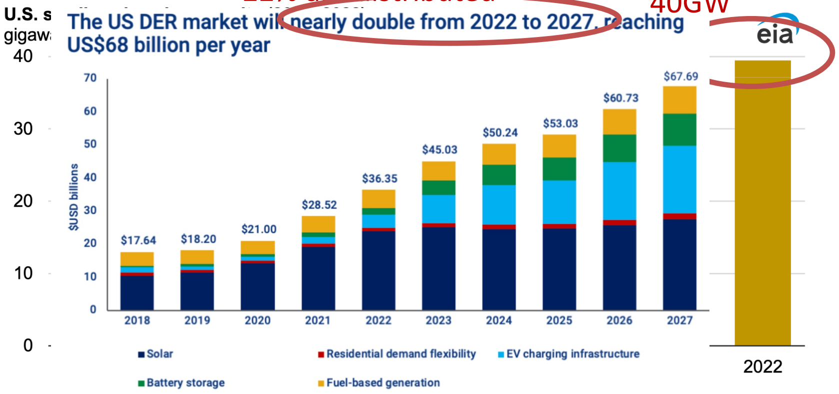
What will you do?

- *Background*
- *Problem formulation*
- *Main Results*
- *Case Study and Conclusion*

Motivation

- United States DER integration increase

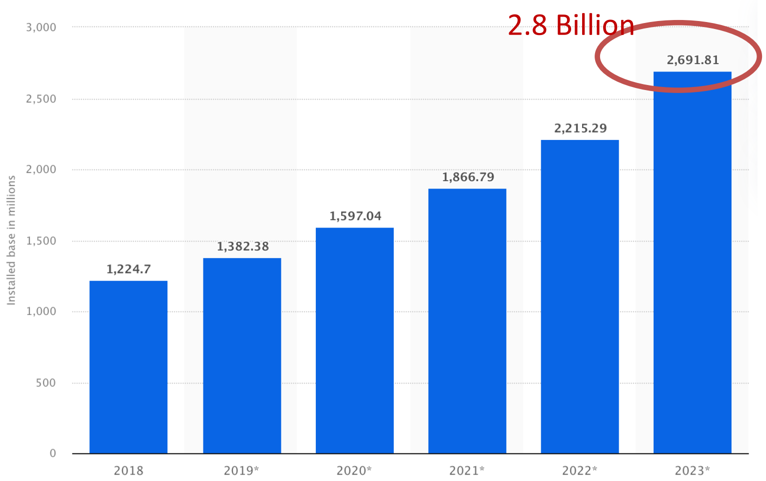
179 GW solar capacity until 2023
22% are distributed
40GW



https://en.wikipedia.org/wiki/Solar_power_in_the_United_States
https://www.eia.gov/todayinenergy/detail.php?id=60341
https://www.woodmac.com/news/opinion/transformation-distributed-energy-resource-market/

- Consumers installed more smart home devices

2.8 Billion

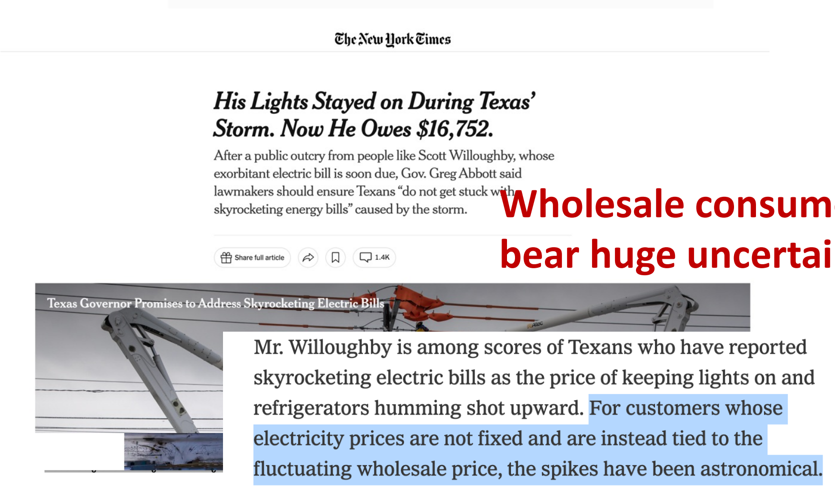


https://www.statista.com/statistics/1075749/united-states-installed-base-of-smart-home-systems/#statisticContainer
https://www.techtarget.com/iotagenda/definition/smart-home-or-building

Consumers become more responsive

Dynamic prices incentivize consumers' responsiveness – uncertainty

- Wholesale markets are inherently uncertain.


The New York Times

His Lights Stayed on During Texas' Storm. Now He Owes $16,752.

After a public outcry from people like Scott Willoughby, whose exorbitant electric bill is soon due, Gov. Greg Abbott said lawmakers should ensure Texans "do not get stuck with skyrocketing energy bills" caused by the storm.

Share full article 1.4K

Texas Governor Promises to Address Skyrocketing Electric Bills

Mr. Willoughby is among scores of Texans who have reported skyrocketing electric bills as the price of keeping lights on and refrigerators humming shot upward. For customers whose electricity prices are not fixed and are instead tied to the fluctuating wholesale price, the spikes have been astronomical.

Wholesale consumers bear huge uncertainty

https://en.wikipedia.org/wiki/Solar_power_in_the_United_States
https://www.eia.gov/todayinenergy/detail.php?id=60341
https://www.woodmac.com/news/opinion/transformation-distributed-energy-resource-market/

- Utility companies adopt dynamic tariffs to incentivize demand responses.

Ameren Power Smart Pricing in Illinois

Retail consumer bear huge uncertainty

https://www.coned.com/en/accounts-billing/your-bill/time-of-use
https://www.srpnet.com/price-plans/residential-electric/time-of-use
https://www.oge.com/wps/portal/ord/residential/pricing-options/smart-hours
https://www.ameren.com/illinois/account/customer-service/bill/power-smart-pricing/

Understand complex risk-aware behaviors facing (price) uncertainty

Background

Literature

- **Data-driven**

https://www.javatpoint.com/group-discussion

Infrastructure

Privacy

Less data-driven previous works

Face limited application problem

- **Model-driven**: Adopt decision-making models with **utility functions** to represent consumers' decision-making process

Quadratic $ax^2 + bx + c$

Piecewise linear

$$e_t = \begin{cases} E & \text{if } \theta_t < v_t(E) \\ v_t^{-1}(\theta_t) & \text{if } v_t(E) \le \theta_t \le v_t(0) \\ 0 & \text{if } \theta_t > v_t(0) \end{cases}$$

Conditional value at risk (CVaR) or robust

$$\mathrm{CVaR}_\alpha(\boldsymbol{X}; z) = \min_{z \in \mathbb{R}} \{z + \frac{1}{1-\alpha} \mathbb{E}\{[\boldsymbol{X} - z]^+\}\}$$

Consumers naturally have high-dimensional and non-linear behavior

Lack of understanding of risk-aversion motivations

Highlight the need for a more sophisticated utility function formulation

Background

What did we do

$$\min_{\mathbf{x},\mathbf{y}_\xi} \quad \mathcal{R}_{\text{cost}}\left[f^S(\mathbf{x}, \mathbf{y}_\xi, \xi)\right]$$

$$\text{s.t.} \quad h^F(\mathbf{x}) = 0, \quad g^F(\mathbf{x}) \leq 0,$$

Risk operator

$$\min_{\mathbf{x},\mathbf{y}_\xi} \quad \mathbb{E}_P\left[f^S(\mathbf{x}, \mathbf{y}_\xi, \xi)\right]$$

Expectation

+ ~~Risk term~~

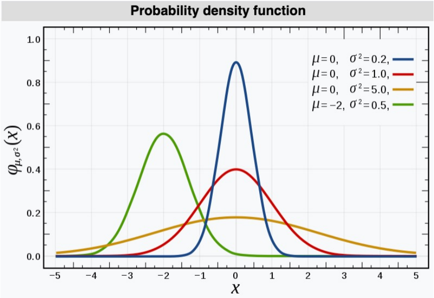
Intentionally

1

2

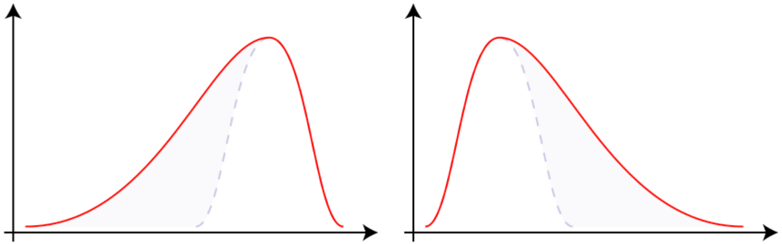
How does the future uncertainty distribution affect the risk-neutral decision-making process?

- Normal distribution – mean, variance



https://en.wikipedia.org/wiki/Normal_distribution

- Skewed (asymmetry) distribution – mean, variance, **shape**



Negative skew

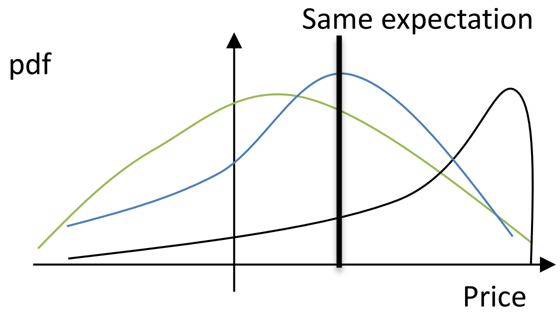
Positive skew

https://en.wikipedia.org/wiki/Skewness

Background

Contribution

- We establish a theoretical framework to model demand behavior to future volatile electricity prices with a constant expectation value. The demand is modeled with a risk-neutral cost-saving objective in a sequential decision-making context;
- We found that demand models with **quadratic cost functions** are **distribution-insensitive;**
- We prove that **super-quadratic cost functions** (higher order than two) result in **prudent demands**;
- We use simulation to verify our results.



English: how consumers respond to future risk

Math-wise: third-order derivative of the utility function

$$\frac{\partial^3 G_t(p_t)}{\partial p_t^3}$$

- *Background*
- *Problem formulation*
- *Main Results*
- *Case Study and Conclusion*

Problem Formulation

Demand model

Discrete time-varying system

Linear system transition

Risk-neutral, cost-saving objective

State cost

$$\min_{p_t} \mathbb{E}_{\Lambda_t} \sum_{t=1}^{T} \left[\lambda_t p_t + C_t(x_t) + G_t(p_t) \right] + V_T(x_T),$$

Energy cost — Action cost — End value function: for value continuity, set to 0

s.t.

$$x_t = Ax_{t-1} + p_t,$$

p_t is non-anticipatory

λ – uncertain price

P – power consumption (battery charging/discharging)

X – state (battery SOC)

***Cost function modeling soft and hard constraints**

Stochastic dynamic programming reformulation

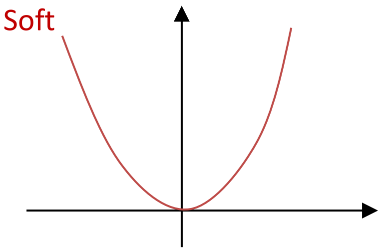
Working backward and recursively solving **a single-stage** optimization for all time *t*

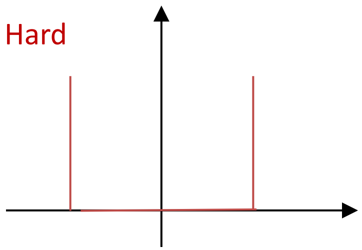
$$Q_{t-1}(x_{t-1}|\lambda_t) = \min_{p_t} \lambda_t p_t + C_t(x_t) + G_t(p_t) + V_t(x_t) \tag{3a}$$

$$V_t(x_t) = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})] \tag{3b}$$

$$\text{s.t. } x_t = Ax_{t-1} + p_t. \tag{3c}$$

Value function: rewards from the future about the current decision, it is a function of time-dependent state value.

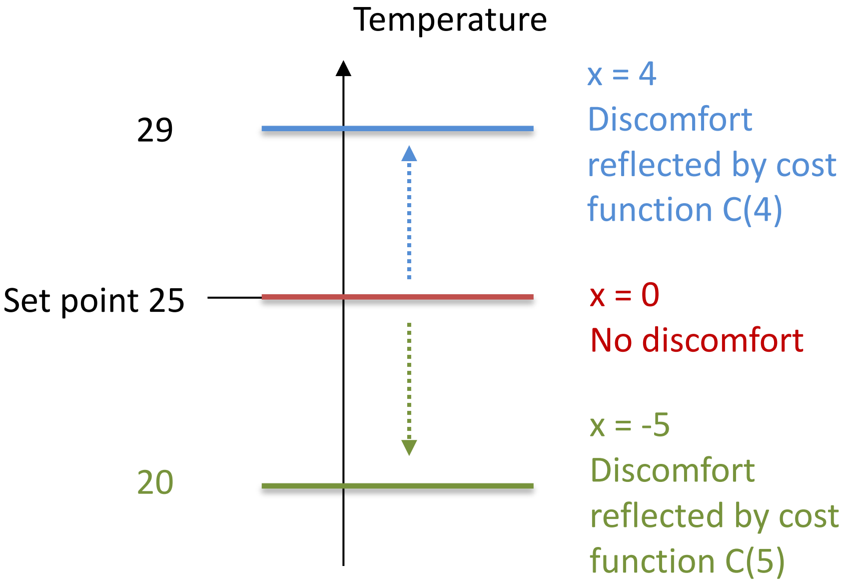




Problem Formulation

Definition - Normalized power and state cost

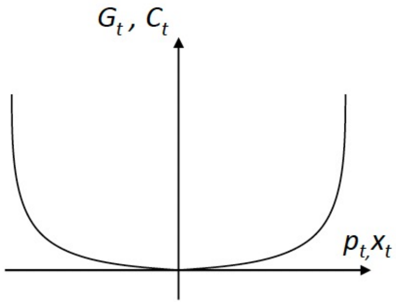
HVAC system (air conditioning)

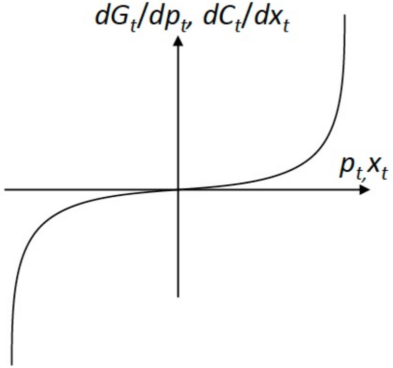


- The system is in equilibrium at zero power and state;
- Deviate from reference (0) increase discomfort (cost);
- Highlight our focus on disturbances and variations.

Definition/assumption

Convex and continuous





- *Background*
- *Problem formulation*
- *Main Results*
- *Case Study and Conclusion*

Main Results

Theorem 1 – Distribution-insensitive demand models

Demand model:

Quadratic action cost

$$Q_{t-1}(x_{t-1}|\lambda_t) = \min_{p_t} \lambda_t p_t + C_t(x_t) + G_t(p_t) + V_t(x_t)$$

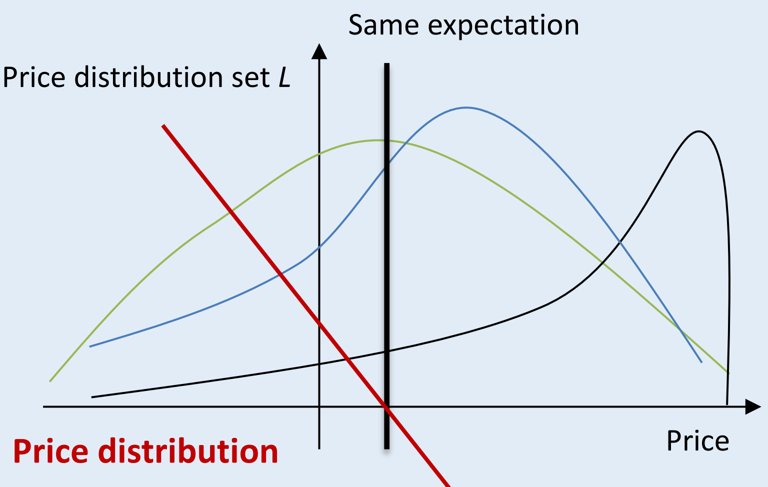
Quadratic State cost

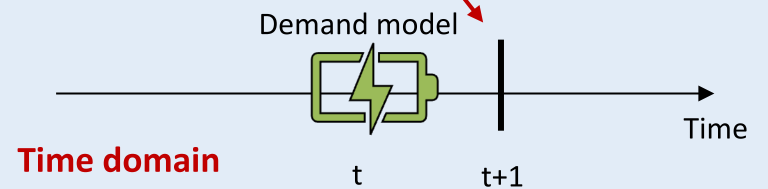
$$V_t(x_t) = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})]$$
$$\text{s.t. } x_t = Ax_{t-1} + p_t.$$

Set end value function to 0

Quadratic function: $G_t(p_t) = \frac{a_p p_t^2}{2}, a_p > 0,$

Demand model





Theorem - **The demand model at time t is distribution-insensitive to price distribution at time t+1**

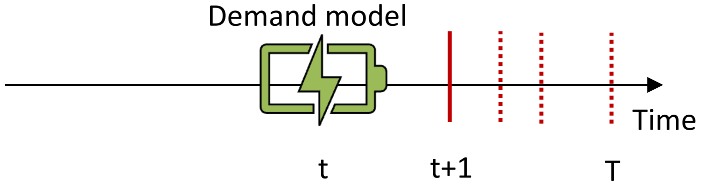
$$\mathbb{E}_{\Gamma_{t+1}}[Q_t(x_t|\lambda_{t+1})] = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})].$$

Value function with different uncertainty distribution

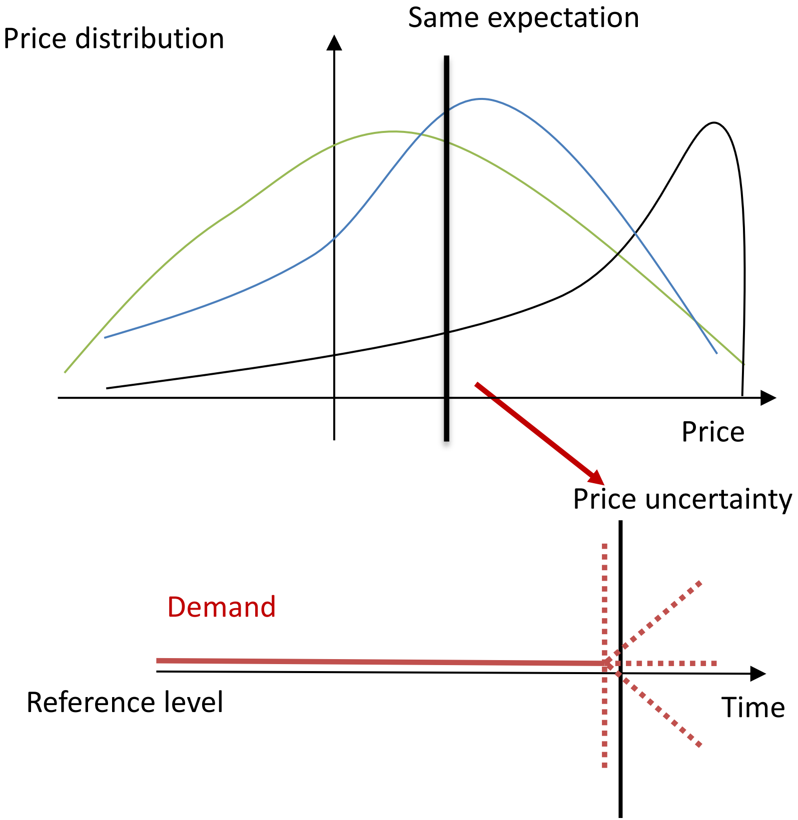
Main Results

Corollary – time extrapolation

Theorem - The demand model at time t is distribution-insensitive to price distribution at time t+1



Distribution insensitive to all future time



Key takeaway

- Demand with quadratic action and state cost function is independent of the future price distribution but only the expectation;

Main Results

Corollary – distribution sensitive demand

Demand model:

$$Q_{t-1}(x_{t-1}|\lambda_t) = \min_{p_t} \lambda_t p_t + C_t(x_t) + G_t(p_t) + V_t(x_t) \quad (3a)$$

$$V_t(x_t) = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})] \quad (3b)$$

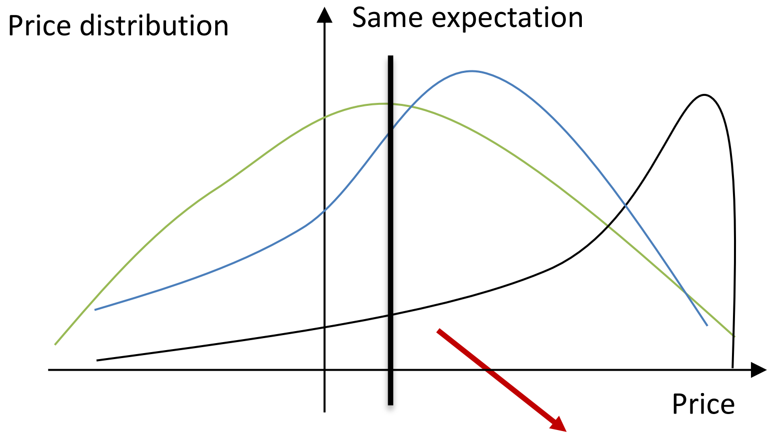
$$\text{s.t. } x_t = Ax_{t-1} + p_t. \quad (3c)$$

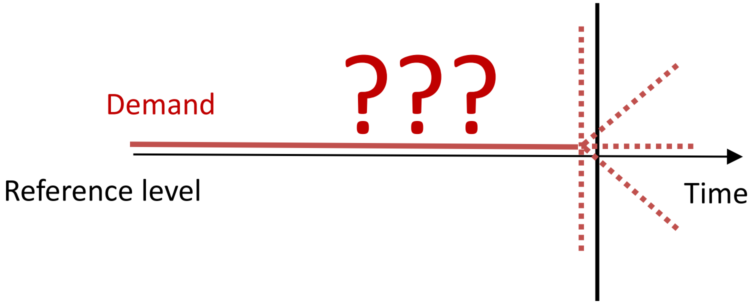
Quadratic function:

$$G_t(p_t) = \frac{a_p p_t^2}{2}, a_p > 0,$$

Super quadratic function:

$$\frac{\partial^3 G_t(p_t)}{\partial p_t^3} \neq 0.$$





Takeaway – problem

Practical situations challenges distribution-insensitive:

- Devices show higher-order cost function performance (thermal comfort and hard constraints)
- Practical price distribution – not symmetrical with zero-mean

Motivated super quadratic prudent formulation

Main Results

Theorem 2 – Prudent demand models

Demand model:

Super Quadratic State cost Set to 0

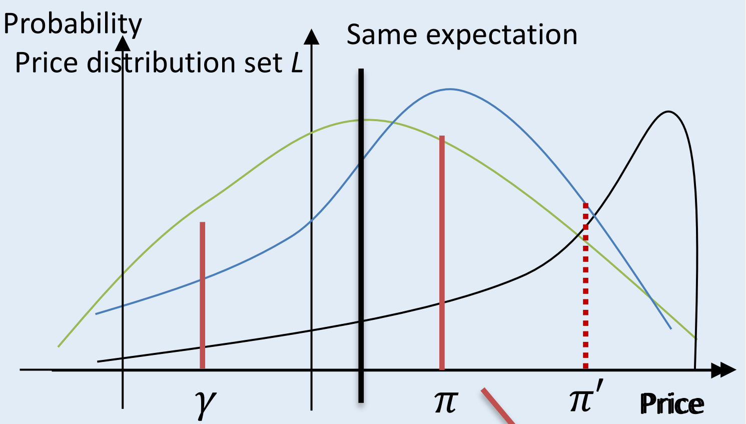
$$Q_{t-1}(x_{t-1}|\lambda_t) = \min_{p_t} \lambda_t p_t + C_t(x_t) + G_t(p_t) + V_t(x_t)$$

Quadratic action cost

$$V_t(x_t) = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})]$$

$$\text{s.t. } x_t = Ax_{t-1} + p_t.$$

Demand model



Price distribution

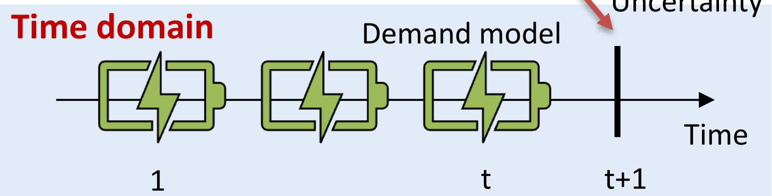
Theorem - The demand model before time t is prudent to price distribution at time t+1

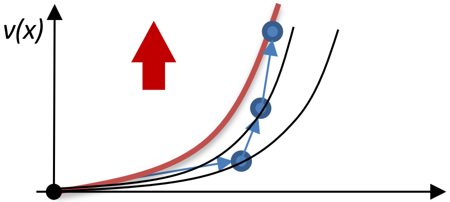
$$\mathbb{E}_{\Gamma_{\tau+1}}[Q_\tau(x_\tau|\lambda_{\tau+1})] \geq \mathbb{E}_{\Lambda_{\tau+1}}[Q_\tau(x_\tau|\lambda_{\tau+1})] \geq Q_\tau(x_\tau|\mathbb{E}_{\Lambda_{\tau+1}}\lambda_{\tau+1}) \geq 0, \forall \tau \leq t$$

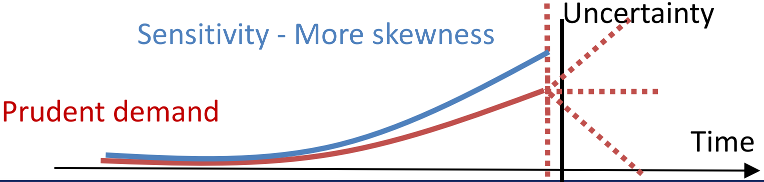
γ, π' Sensitivity Distribution γ, π Expectation

Value function with different uncertainty distribution

Uncertainty

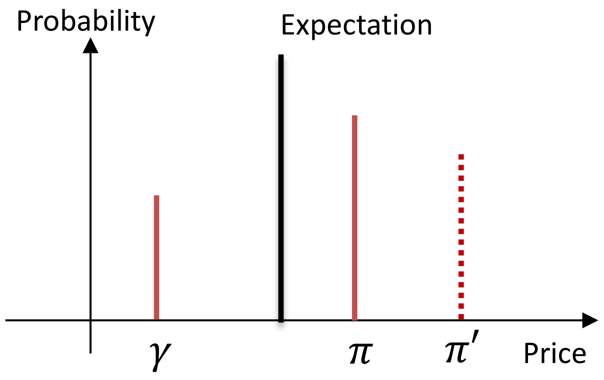


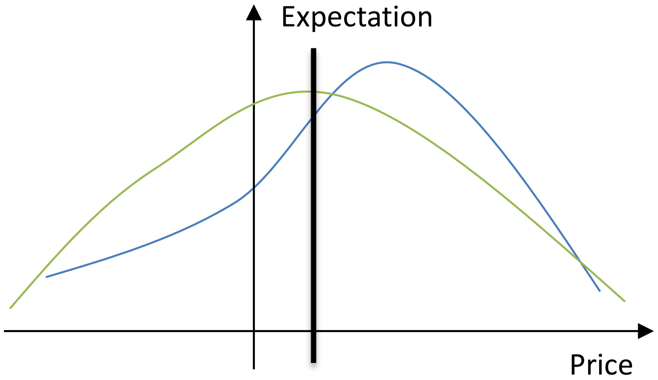




Main Results

Corollary – Distribution & sensitivity extrapolation





Corollary – Strict condition

- Prudent theorem

$$\mathbb{E}_{\Gamma_{\tau+1}}[Q_\tau(x_\tau|\lambda_{\tau+1})] \geq \mathbb{E}_{\Lambda_{\tau+1}}[Q_\tau(x_\tau|\lambda_{\tau+1})] \geq Q_\tau(x_\tau|\mathbb{E}_{\Lambda_{\tau+1}}[\lambda_{\tau+1}]) \geq 0, \forall \tau \leq t$$

- Demand model

$$Q_{t-1}(x_{t-1}|\lambda_t) = \min_{p_t} \lambda_t p_t + C_t(x_t) + G_t(p_t) + V_t(x_t) \quad (3a)$$

$$V_t(x_t) = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})] \quad (3b)$$

$$\text{s.t. } x_t = Ax_{t-1} + p_t. \quad A < 1 \quad (3c)$$

$$x_{\tau_0} \approx A^{t-\tau_0} x_t \quad \to 0$$

- Strict condition

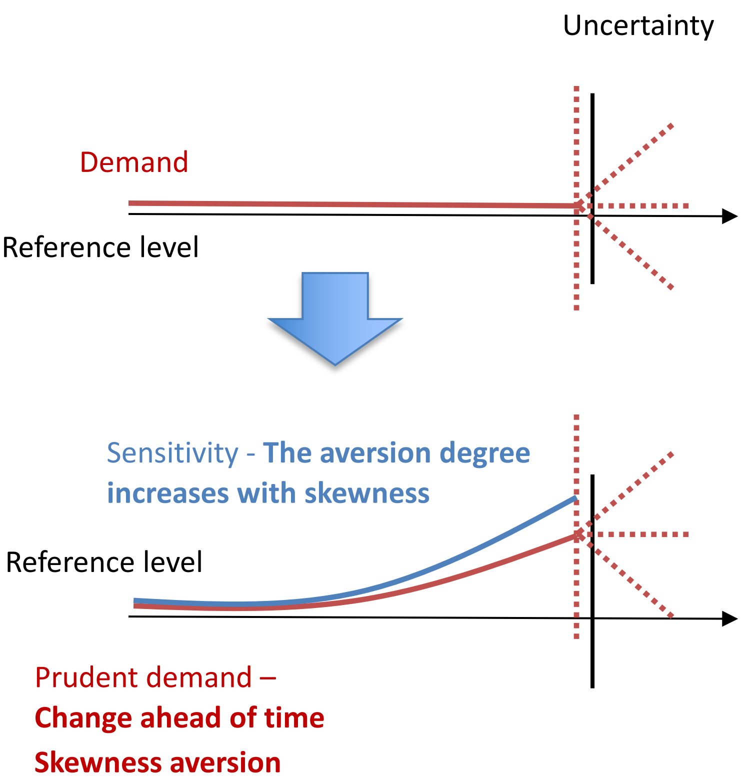
$$\mathbb{E}_{\Gamma_{\tau+1}}[Q_\tau(x_\tau|\lambda_{\tau+1})] > \mathbb{E}_{\Lambda_{\tau+1}}[Q_\tau(x_\tau|\lambda_{\tau+1})] > Q_\tau(x_\tau|\mathbb{E}_{\Lambda_{\tau+1}}[\lambda_{\tau+1}]) > 0, \forall \tau_0 < \tau \leq t.$$

Main Results

Key takeaway

- Prudent demand's value function increases with the future price variance, even with the same expectation;
- The demand level change (aversion) increases with the distribution variance (skewness);
- In discrete cases, e.g., HVAC, possible to show prudence (determined by the cost function parameter and action set)
- Outlier: Symmetrical distribution with the expectation of zero;
- Our results align with the prudence definition from economics.

$$\frac{\partial^3 V_t}{\partial x_t^3} > 0$$



- *Background*
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Case Study

Basic setting

- Quadratic action cost function: $G_t(p_t) = \frac{a_p p_t^2}{2},$
- Log barrier state cost function:

$$C_t(x_t) = -\alpha_c \ln(x_{\max} - x_t) - \alpha_c \ln(x_{\max} + x_t) + 2\alpha_c \ln x_{\max},$$

$$c_t(x_t) = \frac{\alpha_c}{x_{\max} - x_t} - \frac{\alpha_c}{x_{\max} + x_t}$$

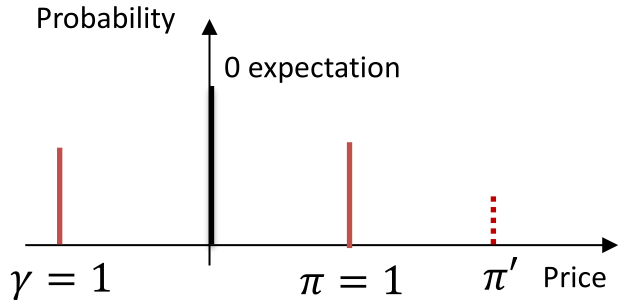
- Parameter:

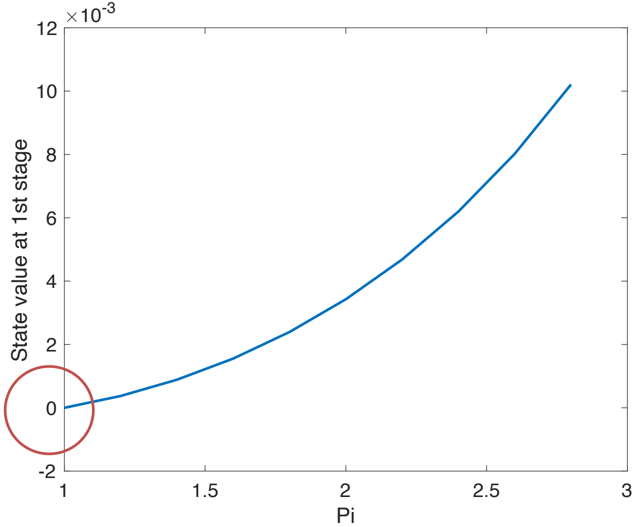
$$\alpha_c = 0.5, A = 1, V_T = 0, a_p = 1, x_{\max} = 20$$

An illustration example

- 2-stage, 2-point price distribution with 0 expectation

$$x_0 = 0, \gamma = -1, \pi = 1 \nearrow$$

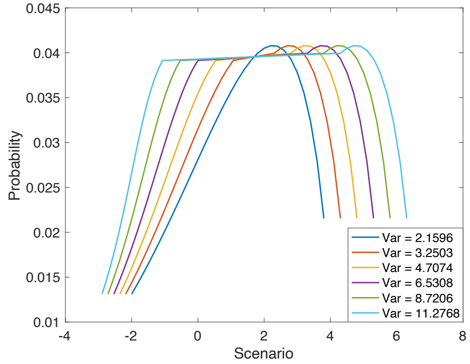




Symmetrical uncertainty with 0 expectation and 0 initial state

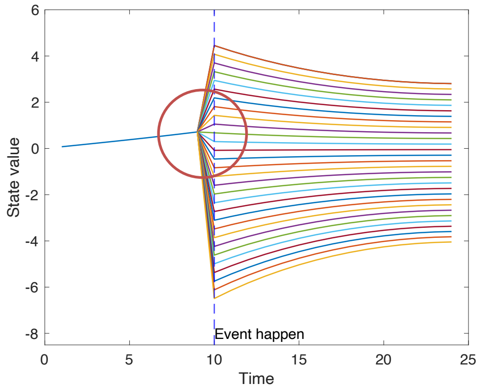
Continuous prudent demand

- 24 stages with 1 interval
- The event happens at the 10^{th} stage
- 6 skewed price distributions with the same expectation and different variance (skewness)



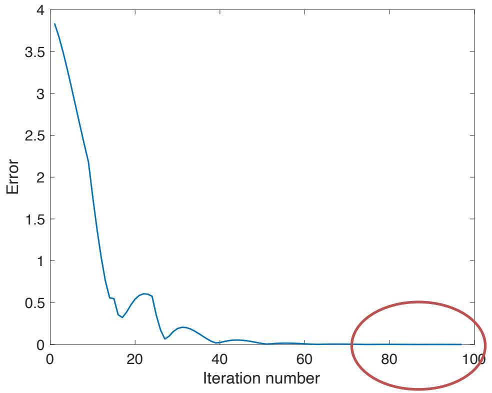
- State under 1^{st} price distribution:

Prudent demand increases before event happen



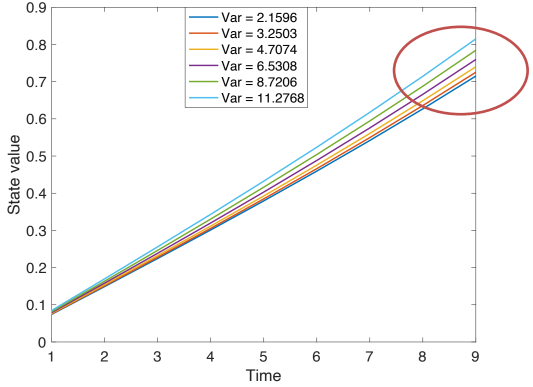
- Convergence under 1^{st} price distribution:

Calculation time: 2s.



- State before 10^{th} with all distributions:

Sensitivity - aversion degree increase



Interpretation and Conclusion

Conclusion

- Provide a theoretical framework to analyze the response behavior of demand to future volatile electricity prices with fixed expectations;
- Quadratic utility/cost formulation results in distribution-insensitive response behavior, i.e., demand's action isn't affected by price distribution, but only by expectation;
- Super quadratic utility/cost formulation results in prudent demand, i.e., demand's action changes ahead of time to respond to the uncertainty, and the change increases with the uncertainty distribution skewness.

Interpretation and Conclusion

Practical implementation

For utility companies or regulators:

- Dynamic pricing tariff mechanism design should consider another demand peak in advance when issuing an incentive-based demand response event for electric vehicles and consumers;

For consumer:

- Bidding strategies for battery or virtual power plants should consider the ‘precautionary saving' behavior.

Selected Reference

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Example

Let's go back to the first example

What is your choice again?

Thanks!

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Q&A

For all details, please reference to - http://arxiv.org/abs/2405.16356