Prudent Price-Responsive Demands

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Background & Research Question

Background:

- DER increases and smart home devices are installed; thus, consumers become more responsive.
- Dynamics prices incentivize consumers' responsiveness, but bring uncertainties.

Key point

- Understand complex risk-aware behaviors of consumers when facing (price) uncertainty;
- Highlight the need for a more sophisticated utility function formulation;
- Intentionally adding risk term instead of natural behaviors.

Research question: How does the future uncertainty distribution affect the risk-neutral decision-making process?

Prudent

Prudent demand - Future price uncertainties affect immediate consumption patterns despite the price expectations remaining unchanged.

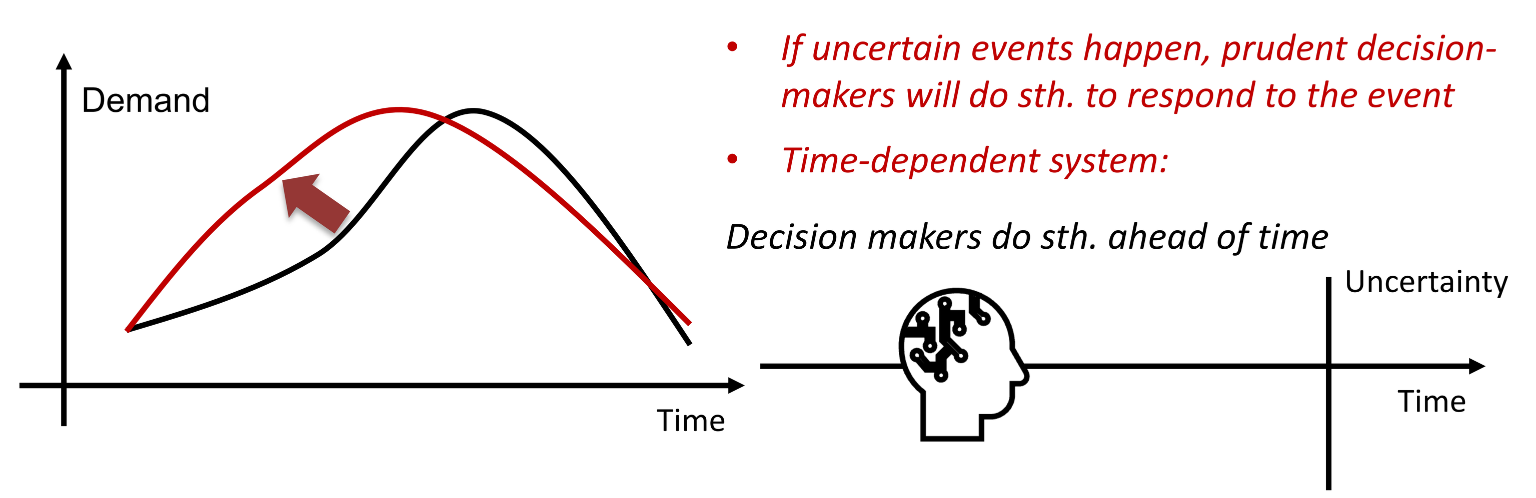


Figure 1. Title explanation.

Model and Formulation

Risk-neutral, cost-saving objective, discrete time-varying linear system $t \in [0, T]$

$$\text{Original } \min_{p_t} \mathbb{E}_{\Lambda_t} \sum_{t=1}^{T} \Big[\lambda_t p_t + C_t(x_t) + G_t(p_t)\Big] + V_T(x_T), \quad (1a)$$

$$\text{Reformulate } Q_{t-1}(x_{t-1}|\lambda_t) = \min_{p_t} \lambda_t p_t + C_t(x_t) + G_t(p_t) + V_t(x_t) \quad (1b)$$

$$V_t(x_t) = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})] \quad (1c)$$

$$\text{s.t. } x_t = A x_{t-1} + p_t. \quad (1d)$$

λ_t - uncertain price following distribution Λ_t; p_t, x_t - demand and state value; $C_t(x_t), G_t(p_t)$ - state cost and action cost function; $Q_{t-1}(x_{t-1}|\lambda_t)$ - action-value function; $V_t(x_t)$ - state-value function; A - discount factor.

Definition 1. *Normalized power and state cost.*

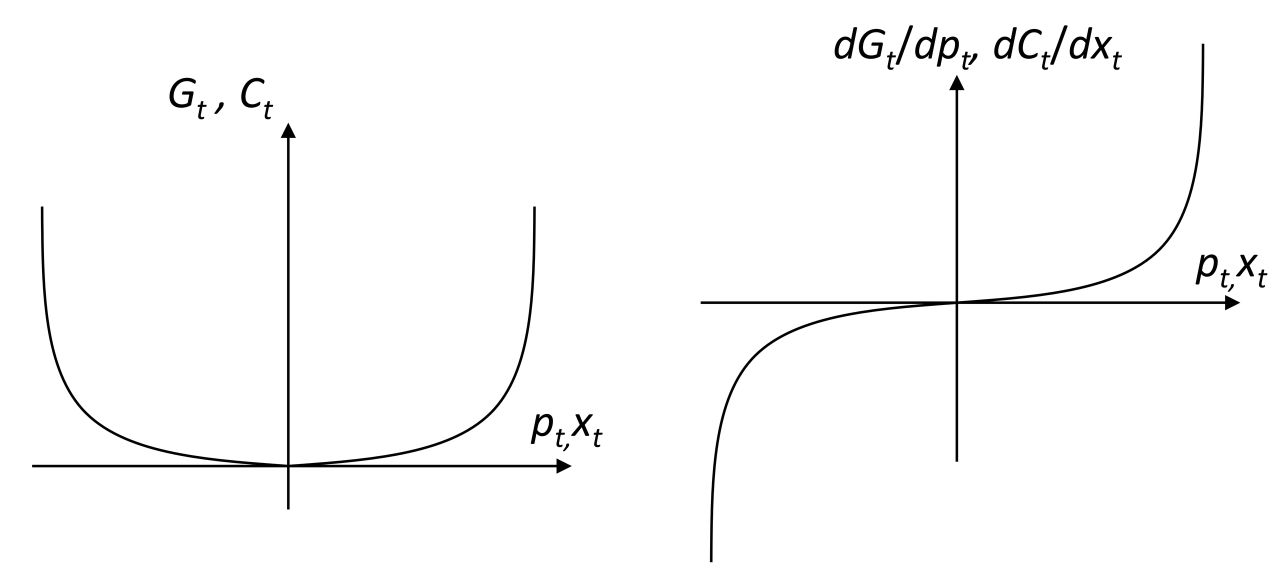


Figure 2. Graph of function C_t, G_t and their derivative.

Key Results

Theorem 1. *Distribution-insensitive demands models.*

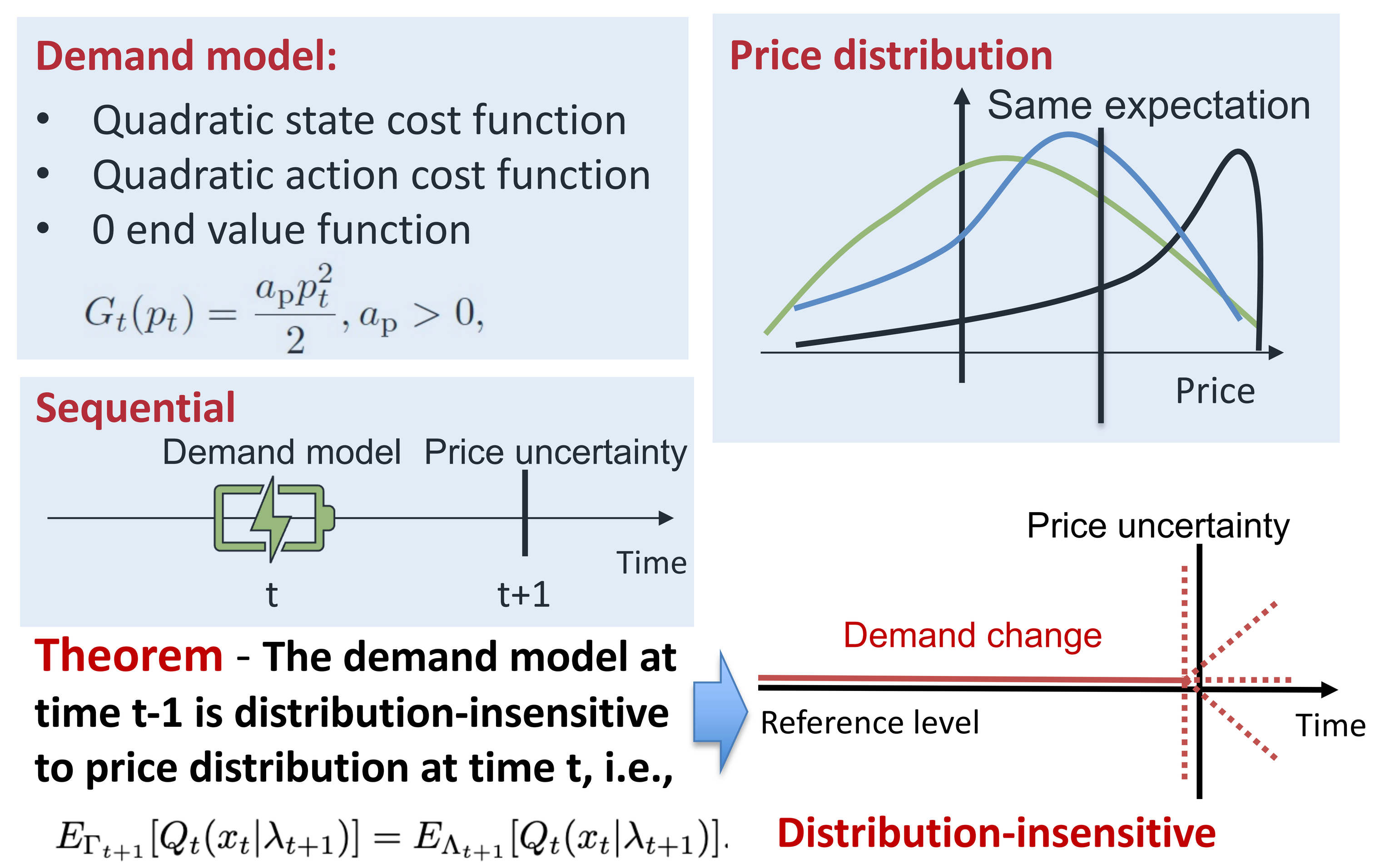


Figure 3. Theorem 1.

Theorem 2. *Prudent demand models.*

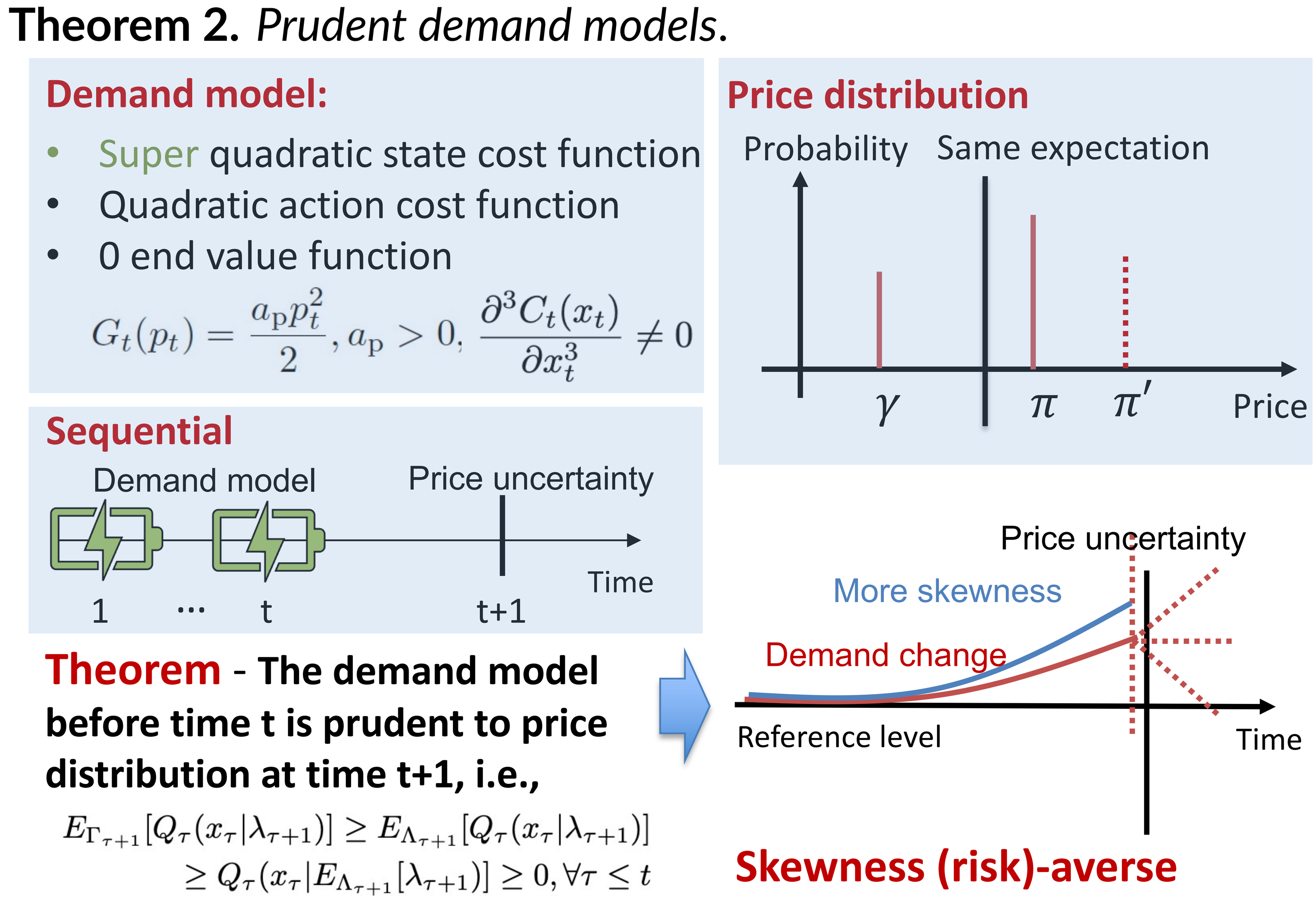


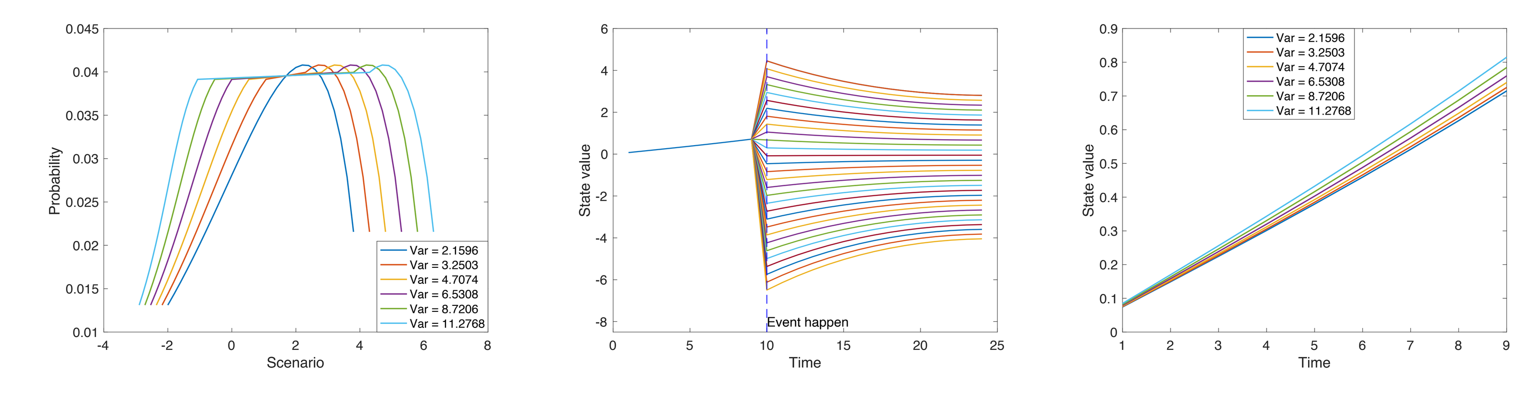
Figure 4. Theorem 2.

We also provide Corollaries about

- *Strict condition*
- *Prudent demand distribution extrapolation*
- *Prudent demand sensitivity extrapolation*

Case Study

Log barrier **State cost** function; Quadratic **Action cost** function

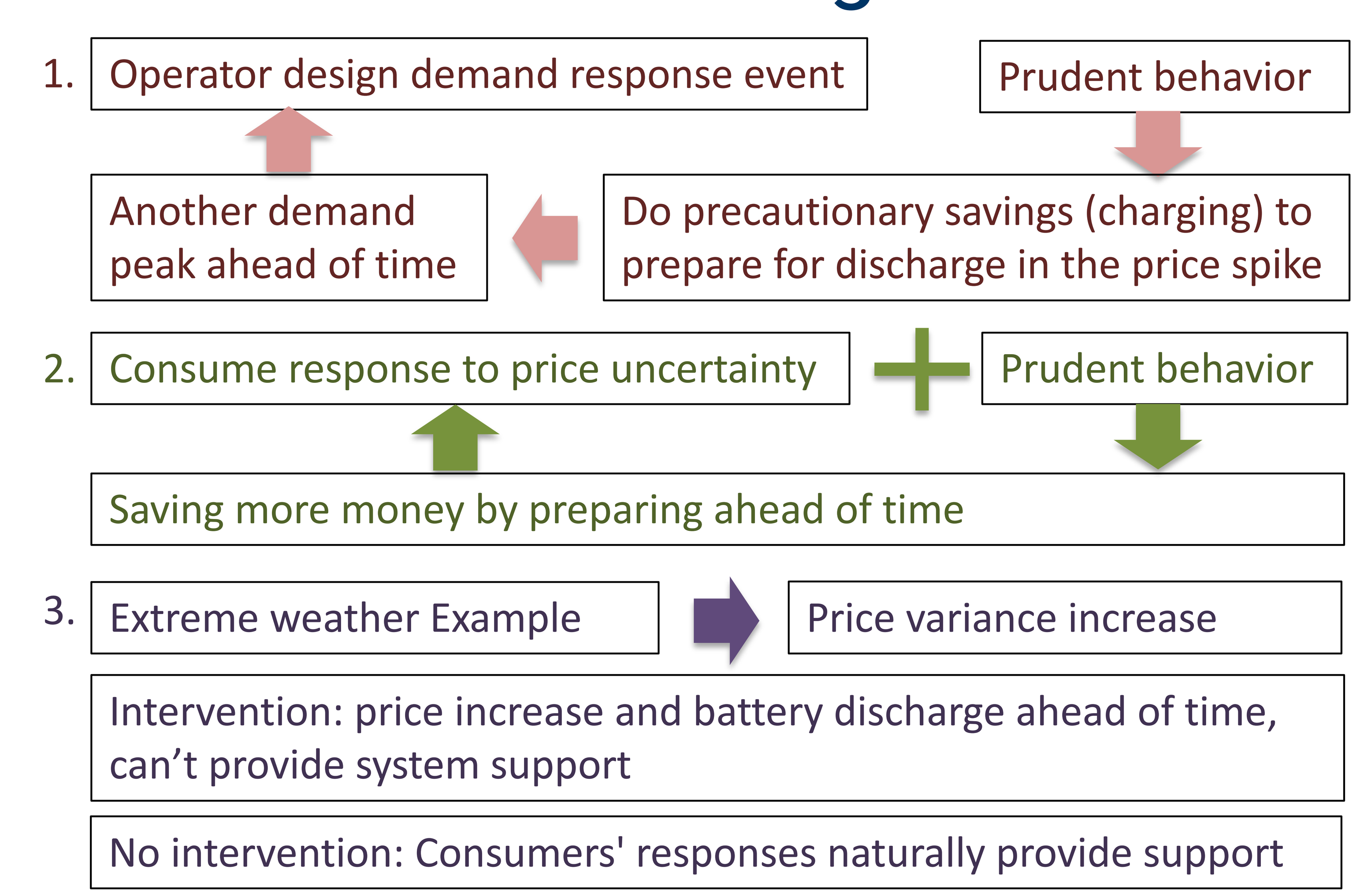


(a) Uncertain price distribution. (b) State value under 1st price distribution (c) State value under all price distributions

Figure 5. Continuous prudent demand results.

- Fig. 5 (b) Skewness (risk)-averse behavior - demand level increases until the event happens.
- Fig. 5 (c) Skewness sensitivity - previous state value and slope increases with the price variance.

Practical insight



References

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