A Stochastic Game Approach for Distributed Voltage Regulation Among Autonomous PV Prosumers

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***Abstract*—The complex voltage variation is an emerging issue caused by the large-scale distributed energy resources (DERs) integration and the forming prosumers. The prosumers with uncertain photovoltaic (PV) generation will serve as autonomous entities for distributed voltage regulation, which have interaction during the voltage regulation process and have not been fully researched. This paper proposed a stochastic game approach for distributed voltage regulation based on autonomous PV prosumers, where the interactive prosumers' PV uncertainties are formulated as a dynamic process during the voltage regulation. An economic incentive-based voltage regulation model is built for autonomous prosumers, which takes the reactive power as strategies and considers the PV curtailment, reactive power compensations, and PV uncertainties. The Markov decision process is adopted, and the approach of changing discrete PV uncertainties to continuous probability distribution is proposed to solve the stochastic game model, while dealing with the "curse of dimensionality" issue arising from the PV uncertainties. Finally, the case study is conducted on the IEEE 33 and 118 node system with real data, and the simulation results demonstrate the effectiveness of the proposed voltage regulation method in terms of the nodal voltage, prosumers' utility, PV uncertainties management, and scalability.**

***Index Terms*—Distributed voltage regulation, stochastic games, PV uncertainty, autonomous prosumer, techno-economic complexities.**

NOMENCLATURE

N, i	Number and index of prosumers
H, h	Number and index of time slot
K, k	Number and index of possible prosumers' actions and photovoltaic (PV) generation scenarios
S_i	Apparent power
$S_{i,\max}$	Maximal apparent power (capacity of PV inverter)
$Q_{0,i,h}$	Initial reactive power
$\Delta Q_{i,h}$	Reactive power strategy
$Q_{i,h,k}$	Reactive power after voltage regulation
$Q_{i,h,\max}$	Upper limit of reactive power regulation
$Q_{i,h,\min}$	Lower limit of reactive power regulation
$P_{\mathrm{PV},i,h}$	PV generation interval of prosumers
$\Delta P_{\mathrm{PV},i,h}$	Curtailment of PV generation
$P_{\mathrm{load},i,h}$	Load demand of prosumers
$P_{i,h,k}$	PV generation after voltage regulation
$p_{i,h,k}$	Probability of uncertain PV generation
$\bar{P}_{\mathrm{PV}} \underline{P}_{\mathrm{PV}}$,	Upper and lower limit of PV generation strategy
$P_{\mathrm{ST},i,H,k}$	State transition probability
$V_{0,i,h}$	Initial node voltage on the grid-connected point
$V_{i,h}$	Node voltage on the grid-connected point after voltage regulation
$V_{i,\min}$, $V_{i,\max}$	Lower limit and upper limit of node voltage, which should equal to 0.95 p.u. and 1.05 p.u. to keep the normal operation
$c_{\mathrm{s},h}$, $c_{\mathrm{b},h}$	Utility grid selling and buying prices
$P_{ij,h}$	Power flow in the branch *ij* between prosumer *i* and *j*
$P_{ij,\min}$, $P_{ij,\max}$	Lower and upper limit of the power flow in branch *ij*
$w_{i,h}$	Prosumers *i*'s contribution for voltage regulation
$d_{i,h}$	Weighting degree of prosumers' reactive power strategy on node voltage
$S_{\mathrm{Q},h,j,i}$	Element of the reactive power-voltage sensitivity matrix
$S_{\mathrm{P},h,i,j}$	Element of the active power-voltage sensitivity matrix
$S_{\mathrm{T},i,h,k}$	State in stochastic game
$A_{i,h,k}$	Action in stochastic game
$P_{\mathrm{ST},i,H,k}$	State transition probability in stochastic game

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I. INTRODUCTION

THE integration of distributed energy resources (DERs), such as rooftop photovoltaic (PV) inverter, in the users' side creates an emerging entity named prosumer, which can consume energy as a consumer and produce energy as a producer [1], [2]. However, the variation of node voltage will be complicated by the prosumers in terms of the bidirectional power flow and fluctuation of PV generation. Unlike the voltage problem in distribution network (DN) without PV inverter, e.g., voltage drop and slowly changing quality, the node voltage in the grid-connected PV system presents both drop and rise, and fast variation [3], thus distribution system operators (DSOs) face new operational challenges.

The DER based voltage regulation is an emerging method that offers different sources of value to the DSO and prosumers by saving the investment of static var compensator (SVC) and allowing the prosumers' high-quality voltage. Research on the DER inverter based-voltage regulation can be grouped into two parts: (i) centralized voltage regulation, and (ii) distributed voltage regulation. The first category primarily focuses on the optimization model, setting the minimal line losses and energy consumption [4], voltage regulation cost [5], and voltage deviation [6] as the optimization goal. The voltage regulation is generally formulated as optimal power flow (OPF) problems. To deal with the non-convex difficulty of power flow calculation, the OPF problem is generally relaxed by the linear approximation of the algebraic power flow equations [7]. The influence of DER's uncertainties is also considered in some studies, because the uncertainties could easily suboptimize the deterministic voltage regulation [8]. Multi-timescale coordinated voltage regulation that considers uncertain voltage fluctuation is modeled as a two-stage stochastic programming problem [9], and the uncertainties can also be considered as probabilistic fashion to use in the chance-constrained voltage regulation [10]. Among these regulation models, each prosumer's reactive power in the DER inverter is the most used strategy for optimizing voltage deviation due to their directly coupled characteristics [4]. The active power curtailment is also an available method because of the larger R/X ratio in DN [11].

With the expansion in the DN and the increased number of prosumers in the voltage regulation, the centralized methods face the problem of heavy computation load and complexity, thus distributed voltage regulation has become a research topic. Some researches aim at solving the centralized optimization problem by a distributed and hierarchical method for supporting the scalability of a large DN [12], e.g., decomposing it into multiple local objectives to implement in the local layer [13], and mimicking the solution from solely locally information based on the data-driven method [14]. Uncertainties of DER have also been considered in the distributed voltage regulation, like building a three-stage robust control method to respond to real-time voltage violation in the third stage by local controllers [15]. For distributed implementation, the DER inverter usually serves as an autonomous entity to regulate voltage on their node, thus, it is inevitable that each inverter has interaction because the voltage variation of each node affects other nodes. This promotes a distributed constraint satisfaction approach for each inverter to update voltage control parameters via interaction with other inverter [16].

As the prosumers with private DER inverter become active participators in the voltage regulation context and can make profits through the process, market mechanism become an effective way for voltage regulation, which is an emerging research topic with techno-economic complexities. The market mechanism for voltage regulation has been designed in [17]–[19], regarding the voltage regulation strategy (i.e., reactive and active power) as the product and ancillary services. With the designing incentive-based value function for DSO and prosumers, DSO can manage each prosumer to participate in the voltage regulation [17]. Since prosumers are autonomous entities to respond to DSO's incentive signal for their voltage regulation, and the variation of each node is coupled, there exists interactions between DSO and each prosumer. Their interaction can be formulated by the game theory, including Nash game between different prosumers with voltage regulation strategy [18], and Stackelberg game between DSO (leader) with price signals and prosumers (followers) with their reactive power strategy response [19].

However, the uncertainties of DER generation are a problem in the market environment. It is noted that the stochastic modeling method of current research mainly focuses on centralized voltage regulation or multi-stage control method, doesn't consider the application in market mechanism and the interaction of autonomous prosumers. The uncertainties of each prosumer may affect their voltage regulation strategies and the whole network voltage, as well as the other prosumers' strategies. To our knowledge, in the market environment, we cannot find research consider the interaction of autonomous prosumers with PV uncertainties in the voltage regulation, which deserves extensive exploration. This complexity presents three challenges: (i) How to formulate the model for prosumers to participate in the voltage regulation with uncertain PV generation, (ii) How to design the distributed voltage regulation mechanism for autonomous prosumers' operations while considering PV uncertainties, and (iii) The effective solution algorithm to face the problem of exponential increment number of solution space caused by the interactive prosumers' PV uncertainties.

To address the aforementioned challenges, the stochastic game theory is introduced. With the concepts of states, probabilities, and state transitions, stochastic game is differs from the static game theory (i.e., non-cooperative game and Stackelberg game) [20], and the interaction of multi-party with uncertainties can be reflected in a dynamic gaming process through state transitions [21], [22]. Therefore, in this paper, the stochastic game approach is proposed for distributed voltage regulation based on autonomous prosumers with PV uncertainties, the contributions are as follows:

1) An incentive-based voltage regulation model is proposed to facilitate prosumers with private PV inverter as autonomous entities to participate in distributed voltage regulation. The proposed model explicitly considers the PV curtailment, reactive power compensations, and uncertain PV generation.

2) A stochastic game model for voltage regulation among autonomous PV prosumers is built based on the proposed voltage regulation model. In the stochastic game model, the interaction of different prosumers in voltage regulation is included, and the PV uncertainties are formulated via dynamic processes.

3) The solution method to the 'curse of dimensionality' of solution space caused by PV uncertainties is proposed, where the Markov decision process (MDP) is adopted, and the approach of continuing discrete PV uncertainties is designed. Then the method is implemented by autonomous PV prosumers in a distributed way.

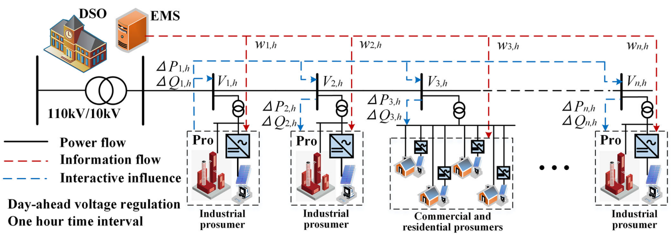

Fig. 1. Voltage regulation framework.

II. Framework

The voltage regulation framework includes prosumers and DSO, which is shown in Fig. 1. The prosumers install the PV inverter, then use this inverter access to the DN nodes and conduct voltage regulation. The PV can generate electricity, and the rest of the PV capacity can be used to keep the voltage in normal operation requirements by regulating the reactive power. 80% of the PV inverter capacity is set to the upper limit of PV generation. The reactive power regulation is based on the inverter, which can operate in two modes: absorb reactive power and generate reactive power. The user energy management system (UEMS) is also equipped in the prosumers to collect load and PV generation data and conduct the calculation for voltage regulation.

The DSO that owns the energy management system (EMS) is in charge of the DN and managing prosumers to participate in the voltage regulation. DSO also gathers the load and PV generation information of all prosumers and calculating the DN parameters. These parameters include power flow and node voltage that should be transmitted to prosumers for distributed voltage regulation.

The market environment is considered in the voltage regulation framework, where prosumers can get profit through participating in the voltage regulation and suffer penalties when voltage deviation. Besides, the active power that can support the prosumers' energy consumption can be traded through the DSO, while the reactive power cannot be traded with other prosumers or the utility grid.

III. System Model Under Stochastic Factors

A. Stochastic Factors in the Voltage Regulation

For the prosumers' PV inverter, the active power P, reactive power Q, and maximal apparent power $S_{\max}$ (capacity of PV inverter) should satisfy the following constraints:

$$P^2 + Q^2 \leq S_{\max}^2 \tag{1}$$

The maximal apparent power $S_{\max}$ is a constant, which is determined by the performance of a certain PV inverter. The voltage regulation is mainly based on reactive power adjustment. However, when $P^2 + Q^2 = S_{\max}^2$, reactive power reaches the maximal adjustment limit, then the active power curtailment is required to provide a wider adjustment range for reactive power. Therefore, as shown in Fig. 2, there are two types of voltage regulation methods in sequence: (i) voltage regulation by reactive power under the condition $P^2 + Q_{\max}^2 = S_{\max}^2$ and

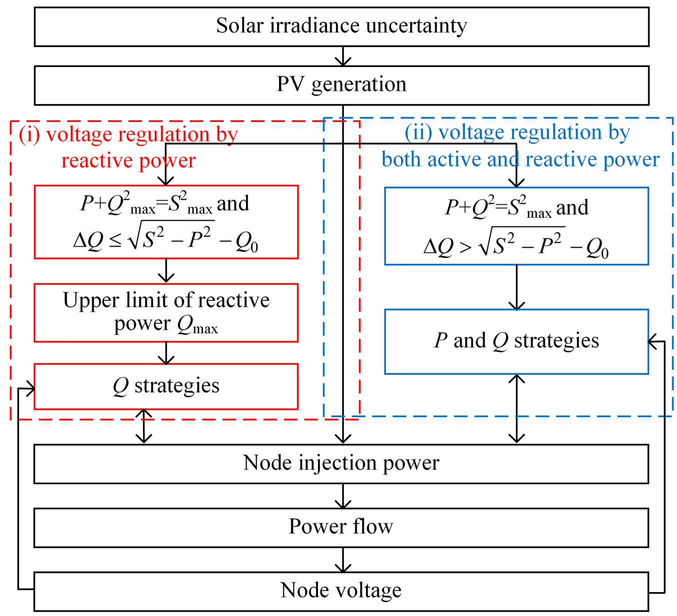

Fig. 2. Voltage regulation method and the influence of PV uncertainties.

$\Delta Q \leq (S_{\max}^2 - P^2)^{\frac{1}{2}} - Q_0)$, (ii) voltage regulation by both active and reactive power under the condition $P^2 + Q^2 = S_{\max}^2$ and $\Delta Q > (S_{\max}^2 - P^2)^{1/2} - Q_0)$.

The prosumers and DN face the uncertainties that come from PV's fluctuation and intermittence, which will change the PV generation. As $S_{\max}$ is a constant, the reactive power Q and active power P will be affected by the uncertain PV generation. Based on the two types of voltage regulation method, Fig. 2 shows the influence of uncertain PV generation in three parts: (i) The reactive power strategy when taking the first voltage regulation method, where the PV uncertainties affect the upper limit of reactive power $Q_{\max}$, then affect the regulation range of reactive power strategy. (ii) The reactive power and active power strategy when taking the second voltage regulation method. In this situation, $P^2 + Q^2 = S_{\max}^2$ is always satisfied, the active power strategy is directly affected by the PV uncertainties, then affects the reactive power strategy. (iii) The power flow in the DN will also be affected. The calculation of power flow requires the injection active and reactive power of each node, which are determined by the prosumers' netload (load demand minus PV generation) and affected by the PV uncertainties.

The node voltage is determined by the power flow calculation with reactive power and active power strategy. These three parts are all affected by the uncertain PV generation. Therefore, the PV uncertainties, active and reactive power strategy, and node voltage are coupled, reflecting the great influence of stochastic factors on the voltage regulation.

The PV uncertainties can be described as a random variable, with the probability distribution determined by the statistical data, which comes from the record of PV generation. The PV uncertainties of prosumer i in time slot h are denoted as $\sigma_{i,h}$, and the distribution can be expressed as follows:

$$\sigma_{i,h} \sim \mathbb{Z}\left(P_{\mathrm{PV},i,h}, p_{i,h}\right), \tag{2}$$

where $\mathbb{Z}(\cdot)$ is the probability distribution obtained by discrete statistical data of PV, the method to get $\mathbb{Z}(\cdot)$ is based on polynomial interpolation or fitting, $P_{\mathrm{PV},i,h}$ is the PV generation interval of prosumer *i* in time slot *h*, and $p_{i,h}$ is the probability that the PV generation level falls in this interval.

B. Incentive-Based Voltage Regulation Model for Prosumers

The prosumer *i* in time slot *h* includes PV production $P_{\mathrm{PV},i,h},\ i \in N,\ h \in H$, load demand $P_{\mathrm{load},i,h}$, initial reactive power $Q_{0,\ i,h}$, and capacity of each PV inverter $S_{i,\max}$. The load demand of each prosumer is assumed as constant during the voltage regulation. It is noted that each prosumer regulates voltage through injecting or absorbing reactive power in their connected node. No matter how many prosumers connect to a node, they all provide reactive power in the same node to adjust the voltage of the DN, i.e., have the same influence on the voltage regulation. Each prosumer can respectively decide their optimal strategies, then these strategies can work together to regulate the node voltage. Therefore, these prosumers that access one node can be equivalent to a comprehensive prosumer (or a coalition) when conducting the voltage regulation, and the number of nodes in the DN can be regarded as equals to the number of prosumers.

To keep the node voltage of DN in the normal operation requirement and improve the voltage quality (i.e., approach to 1 p.u.), two economic incentive-based voltage regulation models are proposed for each prosumer correspond to two types of voltage regulation methods. The model aims at realizing the voltage regulation while maximizing the utility of each prosumer.

1) Voltage Regulation by Reactive Power: When the deviation of node voltage is small (i.e., $\Delta Q_{i,h} \leq (S_{i,\max}^2 - P_{\mathrm{PV},\ i,h}^2)^{1/2} - Q_{0,i,h}$), the voltage regulation model $U_{1,i,h}$ is composed of reactive power compensations, voltage deviation penalty, and inverter losses. which can be expressed as follows:

$$U_{1,i,h}\left(\Delta Q_{i,h}\right) = p_{i,h} * u_{1,i,h},$$

$$u_{1,i,h} = \Big(w_{i,h}\left(a_{i,h}\left|\Delta Q_{i,h}\right| + b_{i,h}\right) - \alpha_{i,h}\left(e^{|1-V_{i,h}|} - 1\right) - \left(e_{i,h}S_{i,h}^2 + f_{i,h}S_{i,h} + r_{i,h}\right)\Big), \quad (3)$$

$$S_{i,h} = \sqrt{\left(Q_{0,\ i,h} + \Delta Q_{i,h}\right)^2 + P_{\mathrm{PV},i,h}^2}\,, \quad (4)$$

where $u_{1,i,h}$ is the payoff without considering the influence of uncertain PV generation.

The first term $w_{i,h}(a_{i,h}|\Delta Q_{i,h}| + b_{i,h})$ is the compensation of participating in voltage regulation, in which the $a_{i,h}$ and $b_{i,h}$ is the coefficient of the compensation, $w_{i,h}$ is prosumers *i*'s contribution for voltage regulation. The compensations are hard to determine by the practical generation, because there is no cost for reactive power generation. The linear type of compensation is widely used to measure the compensations of reactive power [18], [19], which means the prosumers can get more utility when their reactive power adjustment is increased. The parameter $a_{i,h}$ and $b_{i,h}$ is introduced to serve as the liner type function parameter.

Because the reactive power adjustment of each prosumer has different influence on node voltage of the whole DN, the compensations of each prosumer in each node should be distinguished based on their contribution for the voltage regulation. The contribution of prosumer *i* is calculated as follows:

$$w_{i,h} = \frac{d_{i,h}}{\sum_i^N d_{i,h}}\,, \quad (5)$$

where $d_{i,h}$ is the weighting degree, and means the influence of prosumer *i*'s reactive power regulation on all node voltages of the DN. The calculation method is expressed as follows:

$$d_{i,h} = \sum_{j=1}^{N} S_{\mathrm{Q},h,j,i}\,, \quad (6)$$

where $S_{\mathrm{Q},h,j,i}$ is defined as the node voltage of the DN divided by the reactive power adjustment, and the calculation is based on the Jacobi matrix of power flow [7]:

$$S_{\mathrm{Q},i,j}^t = \frac{\Delta V_i^t}{\Delta Q_j^t}\,. \quad (7)$$

The matrix reflects the sensitivity of voltage regulation, i.e., voltage change in node *i* resulted from the reactive power adjustment in node *j*. It is noted that the calculation method of weighting degree $d_{i,h}$ is set by the market manager (DSO), which is fair for all the prosumers to gain profit from the market. The reason is that the degree is determined by the structure and power flow of the DN. Based on the specific DN's structure, if prosumers want to gain more profit, they should contribute more reactive power adjustment to optimize the DN's voltage, while changing the power flow to improve the weighting degree.

The second term $\alpha_{i,h}(e^{|1-V_{i,h}|} - 1)$ is a penalty to judge the severity of voltage deviation, $e^{|1-V_{i,h}|}$ is used to express the penalty, and $\alpha_{i,h}$ is the preference parameter to express the sense of avoiding penalty of the voltage $V_{i,h}$. The more the voltage closes to 1 p.u., the less penalty the prosumer suffers, and the penalty increment will increase with the voltage deviation [18]. The voltage $V_{i,h}$ is calculated by the sensitivity matrix for avoiding multiple power flow calculation:

$$V_{i,h} = V_{0,i,h} - \sum_{j=1}^{N} S_{\mathrm{Q},h,i,j}\Delta Q_{j,h}, \quad (8)$$

where $V_{0,i,h}$ is calculated by the power flow and affected by the PV uncertainties.

The third term is the PV inverter losses during the reactive power regulation, which is arisen from equivalence resistance, switching action, etc. [23]. According to [4], [24], the power loss in an inverter can be approximated by a second polynomial function of its apparent power S_i, where $e_{i,h}$ is the ohmic losses proportional to current squared (I^2), $f_{i,h}$ is the voltage dependent losses over the power electronic components, which is proportional to its current (I). $r_{i,h}$ is the inverter's standby loss.

The adjustment of reactive power is limited by the PV generation, inverter's capacity, and upper and lower adjustment range, which also illustrates the indirect influence of PV uncertainties. Besides, the constraints for the normal operation of DN should also be satisfied, which includes the power flow constraints and node voltage constraints, i.e., the power flow cannot exceed the transmission capacity of the power line, the node voltage should within the upper and lower bounds. The constraints are expressed as follows:

$$P_{\mathrm{PV},\ i,h}^{2}+\left(Q_{0,i,h}+\Delta Q_{i,h}\right)^{2}\leq S_{i,\max}^{2}, \tag{9}$$

$$\Delta Q_{i,h,\min}\leq\Delta Q_{i,h}\leq\Delta Q_{i,h,\max}, \tag{10}$$

$$V_{i,\min}\leq V_{i,h}\leq V_{i,\max}, \tag{11}$$

$$P_{ij,\min}\leq P_{ij,h}\leq P_{ij,\max}, \tag{12}$$

where $P_{ij,\min}$ and $P_{ij,\max}$ are negative value and positive value, and the symbol means the direction of power flow.

It is stress that these three terms in the voltage regulation model (3) is affected by the PV uncertainties $p_{i,h}$. The reason is that the uncertain PV generation affects the node voltage and the reactive power regulation constraints (9-12), while the reactive power regulation strategies are decided by these two factors.

2) Voltage Regulation by Both Active and Reactive Power: When the high PV output led to a large voltage deviation (i.e., $\Delta Q_{i,h}>(S_{i,\max}^{2}-P_{\mathrm{PV},\ i,h}^{2})^{1/2}-Q_{0,i,h}$), the voltage regulation cannot be realized only by the reactive power. The PV generation (i.e., active power) curtailment should be considered to provide more regulation range for the reactive power. The voltage regulation model in this condition $U_{2,i,h}(\Delta Q_{i,h},\Delta P_{\mathrm{PV},i,h})$ is composed of reactive power compensations, voltage deviation penalty, inverter losses, and PV generation curtailment, which is expressed as follows:

$$U_{2,i,h}\left(\Delta Q_{i,h},\Delta P_{\mathrm{PV},i,h}\right)=p_{i,h}*u_{2,i,h},$$

$$\begin{aligned}u_{2,i,h}=&\left(w_{i,h}\left(a_{i,h}\left|\Delta Q_{i,h}\right|+b_{i,h}\right)-\alpha_{i,h}\left(e^{\left|1-V_{i,h}\right|}-1\right)\right.\\&-\left.\left(e_{i,h}S_{i,h}^{2}+f_{i,h}S_{i,h}+r_{i,h}\right)\right)\\&-c_{s,h}\min(\Delta P_{\mathrm{PV},i,h},P_{\mathrm{PV,i,h}}-P_{\mathrm{load},i,h})-c_{b,h}\\&\max(\Delta P_{\mathrm{PV},i,h}-(P_{\mathrm{PV},i,h}-P_{\mathrm{load},i,h}),0)\end{aligned}, \tag{13}$$

$$S_{i,h}^{2}=\left(P_{\mathrm{PV},i,h}-\Delta P_{\mathrm{PV},i,h}\right)^{2}+\left(Q_{0,i,h}+\Delta Q_{i,h}\right)^{2}=S_{i,\max}^{2}, \tag{14}$$

where $u_{2,i,h}$ is the payoff with active and reactive power regulation without considering the influence of uncertain PV generation, the utility grid selling and buying prices $c_{s,h}$ and $c_{b,h}$ is set based on the feed-in tariff of some countries, and $\Delta P_{\mathrm{PV},i,h}$ is calculated by:

$$\left(P_{\mathrm{PV},i,h}-\Delta P_{\mathrm{PV},i,h}\right)^{2}+\left(Q_{0,i,h}+\Delta Q_{i,h}\right)^{2}=S_{i,\max}^{2}, \tag{15}$$

$$\Delta P_{\mathrm{PV},i,h}=P_{\mathrm{PV},i,h}-\sqrt{S_{i,\max}^{2}-\left(Q_{0,i,h}+\Delta Q_{i,h}\right)^{2}}. \tag{16}$$

The calculation method of node voltage is also different with the first types (8) due to the PV curtailment, which is expressed as:

$$V_{i,h}=V_{0,i,h}-\sum_{j=1}^{N}S_{\mathrm{P},h,i,j}\Delta P_{\mathrm{PV},j,h}-\sum_{j=1}^{N}S_{\mathrm{Q},h,i,j}\Delta Q_{j,h}, \tag{17}$$

where the calculation method of $S_{\mathrm{P},h,i,j}$ is the same to $S_{\mathrm{Q},h,i,j}$.

Although the expression of PV inverter losses is the same as the first type (3), the calculation method for the apparent power is different, which is a constant in this situation. The reason is that once the active power curtailment is considered, the reactive power adjustment has reached the maximum limit. More adjusting range can only be obtained through curtailing active power. Therefore, the apparent power always equals the maximum limit, i.e., the capacity of the PV inverter.

The *min* term is the cost of reducing PV generation, which is originally sold to the utility grid to make profits. The *max* term is the cost of purchasing active power to satisfy the load demand of each prosumer. These two terms only exist in the second voltage regulation method. The reason is that only the active power can participate in the energy trading with the utility grid, and the reactive power cannot be traded. This voltage regulation model $U_{2,i,h}(\Delta Q_{i,h})$ is more affected by the PV uncertainties than $U_{1,i,h}(\Delta Q_{i,h})$. Other than the indirect influence, the PV uncertainties have direct effect on reactive power regulation though (15), and PV curtailment through (16).

The constraints of the voltage regulation model $U_{2,i,h}(\Delta Q_{i,h})$ also include the upper and lower bounds of $\Delta Q_{i,h}$ and DN's operation constraints about power flow $P_{ij,h}$ and node voltage $V_{i,h}$ (i.e., Eqs. (9-12)), as well as the constraints of PV curtailment $\Delta P_{\mathrm{PV},i,h}$:

$$\underline{P}_{\mathrm{PV}}\leq\Delta P_{\mathrm{PV},i,h}\leq\bar{P}_{\mathrm{PV}}. \tag{18}$$

C. Stochastic Game Model With Interactive PV Uncertainties for Voltage Regulation

The prosumers are coupled in the voltage regulation process because the voltage of different nodes is coupled in the DN, i.e., one node reactive power change will affect the node voltage of the DN. This kind of problem that interactive multi-party with their own voltage regulation goal can be formulated by game theory. Based on the influence of PV uncertainties, multiple system states are generated in different time slots, and the voltage regulation environments are dynamic in the time horizon. Therefore, the stochastic game model with a multi-stage game process is built to formulate the interactive multi-prosumer with PV uncertainties, where the states transition happens between different stages due to the prosumers' action in each time slot:

$$G=\left\langle N,\boldsymbol{S}_{\mathrm{T},h},\boldsymbol{A}_{h},\boldsymbol{P}_{\mathrm{ST},h}\left(A_{i,h,k}S_{\mathrm{T},i,h-1}\right),\boldsymbol{U}_{h},h\in H\right\rangle, \tag{19}$$

$$S_{\mathrm{T},i,h,k}=\left\{Q_{i,h,k},P_{i,h,k},V_{i,h,k}\right\},S_{\mathrm{T},i,h,k}\in\boldsymbol{S}_{\mathrm{T},h}, \tag{20}$$

$$Q_{i,h,k}=Q_{0,i,h,k}+\Delta Q_{i,h,k} \tag{21}$$

$$P_{i,h,k}=P_{\mathrm{PV},i,h,k}-\Delta P_{\mathrm{PV},i,h,k} \tag{22}$$

$$A_{i,h,k}=\left\{\Delta Q_{i,h,k}\right\},A_{i,h,k}\in\boldsymbol{A}_{h}, \tag{23}$$

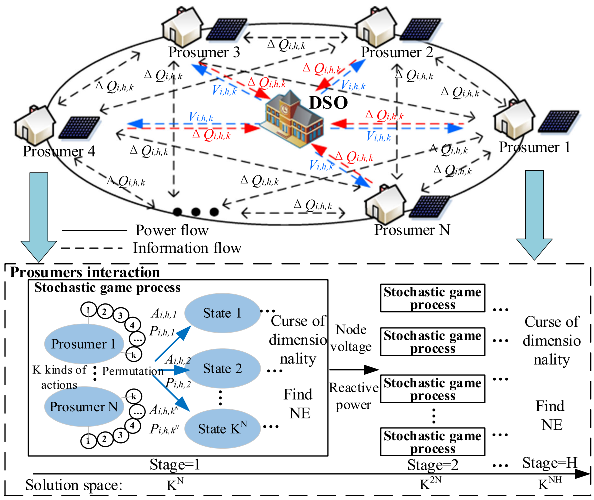

Fig. 3. The stochastic game process.

where the game is implemented hourly-ahead because prosumers set their actions hourly according to the time slot of the voltage regulation, so that the index of the game stage is the same as the time slots. N is the set of prosumers, and they are the players of the game. $\boldsymbol{S}_{\mathrm{T},h}$ is the state set, which includes reactive power, PV generation after voltage regulation, and node voltage. $\boldsymbol{A}_h$ is the action set, which is the reactive power adjustment. In the uncertain PV generation environment, prosumers have multiple possible actions to respond to the different PV generations. Therefore, the number of the actions is equaled to the number of uncertain PV generations, and denoted by $k, k \in K$. The state transition probability $\boldsymbol{P}_{\mathrm{ST},h}(A_{i,h,k}|S_{\mathrm{T},i,h-1})$ is the probability that a prosumer chooses an action in a state, which is determined by the occurrence probability of the PV generation $p_{i,h,k}$. The state transition will result in the state in a game stage change to the other states in the next stage.

$$P_{\mathrm{ST},i,H,k}(Q_{i,H,k}|S^*_{\mathrm{T},i,H-1}) = \frac{p_{i,H,k}\cdot\prod_{h=1}^{H-1}p_{i,h,k}}{\prod_{h=1}^{H-1}p_{i,h,k}} = p_{i,H,k}. \tag{24}$$

where $S^*_{\mathrm{T},i,h-1}$ is the optimal state in the previous stage.

$\boldsymbol{U}_h$ is the profit set of prosumers, which is also the optimization goal of each prosumer in the game model. The profit is calculated based on the voltage regulation model of each prosumer (3), (13). However, it is noted that the model is affected by both self PV uncertainties and other prosumers' PV uncertainties:

$$U'_{1,i,h} = U_{1,i,h}\prod_{j=1,j\neq i}^{N}\frac{p_{j,h,k}K}{\sum_{k=1}^{K}p_{j,h,k}} \tag{25}$$

$$U'_{2,i,h} = U_{2,i,h}\prod_{j=1,j\neq i}^{N}\frac{p_{j,h,k}K}{\sum_{k=1}^{K}p_{j,h,k}} \tag{26}$$

where $\sum_{k=1}^{K}p_{j,h,k}/K$ is the mean value of prosumer i's PV probability for avoiding the reduction in utilities caused by multiplying multiple probabilities less than 1.

Fig. 3 shows the gaming process with state transition and prosumers' optimization. The state set increases with the game occurring in an exponential form, i.e., the number of states in the next stage is K times than previous stage, because one state will change to multiple possible states with the possible actions (arisen from PV uncertainties) and state transition probabilities. Each prosumer optimizes their voltage regulation strategies based on (3, 13) in each stage, while considering the interaction with other prosumers and the influence of other prosumers' PV uncertainties. Then the optimal results for all game players in one stage can be obtained, which is called the Nash equilibrium (NE) [25]. After achieving the NE in the current stage, the game moves to the next stage with the chosen optimal action and corresponding state transition probability.

D. Nash Equilibrium

The NE is the only one feasible solution of the game in each stage.

Definition 1: The NE of prosumer i in time slot h is a set of optimal action $A^*_{i,h} = \{\Delta Q_{i,h}|h = 12,\ldots,H\}$, accompanied with an optimal active power adjustment, power flow and node voltage, if and only if the following expression is satisfied in their own domain:

$$\begin{aligned}
&u_{1,i,h}\left(A^*_{1,h},\ldots A^*_{N,h}\right) \geq u_{1,i,h}\\
&\quad\left(A^*_{1,h},\ldots,A^*_{i-1,h},A^*_{i,h},A^*_{i+1,h},\ldots A^*_{N,h}\right)\\
&u_{2,i,h}\left(A^*_{1,h},\ldots A^*_{N,h}\right) \geq u_{2,i,h}\\
&\quad\left(A^*_{1.h},\ldots,A^*_{i-1.h},A^*_{i.h},A^*_{i+1.h},\ldots A^*_{N.h}\right)\\
&\forall i\in N, h\in H, A^*_{i,h}\in \boldsymbol{A^h}.
\end{aligned}\tag{27}$$

Theorem 1: There always exists a unique NE in each stage in the proposed stochastic game G.

Proof: To demonstrate the uniqueness of the NE in the stochastic game, it has to be demonstrated that the optimal prosumer's strategy is unique, and the optimization goal of prosumers is strictly concave for the reactive power strategy. The possible active power strategy can be expressed by (16). Considering the voltage regulation model has absolute value item $e^{|1-V_{1,i,h}|}-1$, the feasible region of prosumers strategy $\Delta Q_{i,h}$ can be divided into two parts: D_1 and D_2. If $1-V_{i,h}\leq 0$, then $\Delta Q_{i,h}\in D_1$. Otherwise, if $1-V_{i,h}>0$, then $\Delta Q_{i,h}\in D_2$. Finally, these two regions satisfy the following condition:

$$\begin{aligned}
&D_1\cup\ D_2 = D,\\
&D_1\cap\ D_2 = \emptyset.
\end{aligned}\tag{28}$$

For the region D_1, the concave of optimization function with respect to $\Delta Q_{i,h}$ can be illustrated by the Hessian matrix.

For the first type voltage regulation model, i.e., by reactive power, the Hessian matrix H_1 of $U_{1,i,h}$ with respect to $\Delta Q_{i,h}$ is:

$$H_1 = h_1 * \prod_{j=1}^{N}\frac{p_{j,h,k}K}{\sum_{k=1}^{K}p_{j,h,k}},$$

$$h_1 = -\alpha_{i,h} S^2_{Q,h,i,j} e^{V_{i,h}-1} - 2e_{i,h} - f_{i,h}\left(\left(Q_{0,\ i,h} + \Delta Q_{i,h}\right)^2 + P^2_{\mathrm{PV},i,h}\right)^{-3/2} \cdot P^2_{\mathrm{PV},i,h} \tag{29}$$

where $\alpha_{i,h} > 0$, $e^{V_{1,i,h}-1} > 0$, $e_{i,h} > 0$, $f_{i,h} > 0$, $p_{j,h,k} > 0$. Then, we can get that $H_1 < 0$. Therefore, $U_{1,i,h}$ is strictly concave with respect to $\Delta Q_{i,h}$ in the region D_1.

For the second voltage regulation model, i.e., by both active and reactive power, the Hessian matrix H_2 of $U_{2,i,h}$ with respect to $\Delta Q_{i,h}$ is

$$H_2 = h_2 * \prod_{j=1}^{N} \frac{p_{j,h,k} K}{\sum_{k=1}^{K} p_{j,h,k}},$$

$$h_2 = -\alpha_{i,h}\left[\left(\frac{dV_{i,h}}{d\Delta Q_{i,h}}\right)^2 + \frac{S_{P,h,i,j} S^2_{i,\max}}{\left(S^2_{i,\max} - \left(Q_{0,i,h} + \Delta Q_{i,h}\right)^2\right)^{\frac{3}{2}}}\right] e^{V_{i,h}-1} - (c_{s,h} + c_{b,h}) \times \frac{S^2_{i,\max}}{\left(S^2_{i,\max} - \left(Q_{0,i,h} + \Delta Q_{i,h}\right)^2\right)^{\frac{3}{2}}}, \tag{30}$$

where $c_{\mathrm{s},h} > 0$, $c_{\mathrm{b},h} > 0$. Then, the second and third term of h_2 is always positive, and $H_2 < 0$. Therefore, $U_{2,i,h}$ is strictly concave with respect to $\Delta Q_{i,h}$ in the region D_1.

Similarly, $U_{1,i,h}$ and $U_{2,i,h}$ are the strictly concave function with respect to $\Delta Q_{i,h}$ in the region D. Therefore, the prosumers' unique optimal strategies exist in D_1 and D_2, which is denoted by $\Delta Q^*_{i,h,D_1}$ and $\Delta Q^*_{i,h,D_2}$, respectively. Then, the optimal strategies of prosumer *i* in time slot *h* and region *D* can be calculated by:

$$\Delta\ Q^*_{i,h} = \ \mathrm{argmax}(U_{1,i,h}\left(\Delta Q^*_{i,h,D_1}\right), U_{2,i,h}\left(\Delta Q^*_{i,h,D_1}\right), U_{1,i,h}\left(\Delta Q^*_{i,h,D_2}\right), U_{2,i,h}\left(\Delta Q^*_{i,h,D_2}\right)). \tag{31}$$

Thus, there exists a unique SE and ***Theorem 1*** is proved.

IV. Solution Algorithm and Stationery NE

A. Solutions to Curse of Dimensionality

From Fig. 3, it is clear that the number of states increases exponentially with the game stage (i.e., time slot), indicating the game faces the problem of "curse of dimensionality", and making it mathematically unsolvable. The existence of this problem can be concluded into two aspects: (i) increase with the stage, (ii) increase with the number of prosumers and actions. The first part is because *K* possible actions will cause *K* possible states in the first stage for a prosumer, K^2 possible states in the next stage, and K^H in the *H* stage. The second part is because the number of possible states will increase with the number of possible actions *K* and prosumers *N* in one stage, expressed as K^N number of states for *N* prosumers. Combined these two aspects, there will be K^{NH} number of states for *N* prosumers with *K* actions in state *H*, which cannot be directly solved by the mathematical methods.

Based on the observable system states, the first problem can be solved by transferring the stochastic game process to an MDP [26], which includes the states, actions, state transition probability, and profit. The MDP also satisfies the optimality principle of dynamic programming [27], i.e., the sub-result of the optimal results is always optimal, and called 'without aftereffect'. All the game players (prosumers) in one stage can be regarded as a cluster, and each prosumer gets their optimal strategies (NE) and profits in the first stage through their iteration inside the cluster. Then optimal results of the next stage are based on the optimal results in the previous stage (i.e., the sub-result of the optimal results). Therefore, the overall equilibrium can be obtained through the segmented optimization in each stage, and the maximum dimension can be limited within K^N in the process.

The second problem can be solved by continuing the discrete PV generation uncertainties. The continuous probability distribution function of PV uncertainties can be got through continuing the statistical data, where the method includes fitting and statistical regularity. After the continuous process, the discrete probability in voltage regulation model (3), (13) will change to a continuous function with a variable. To realize the convergence of multiple stochastic scenarios, the Euclidean distance between each PV generation scenario is added in the optimization problem, and the new voltage regulation model is expressed as:

$$U'_{1,i,h}\left(\Delta Q_{i,h}\right) = u_{1,i,h} \prod_{j=1}^{N} \frac{g\left(P_{\mathrm{PV}i,j,h}\right)\left(\bar{P}_{\mathrm{PV}} - \underline{P}_{\mathrm{PV}}\right)}{\int_{\underline{P}_{\mathrm{PV}}}^{\bar{P}_{\mathrm{PV}}} g\left(P_{\mathrm{PV}i,j,h}\right)} + \mathrm{g} * \rho_{i,h} \tag{32}$$

$$U'_{2,i,h}\left(\Delta Q_{i,h}, \Delta P_{\mathrm{PV},i,h}\right) = u_{2,i,h} \prod_{j=1}^{N} \frac{g\left(P_{\mathrm{PV},i,j,h}\right)\left(\bar{P}_{\mathrm{PV}} - \underline{P}_{\mathrm{PV}}\right)}{\int_{\underline{P}_{\mathrm{PV}}}^{\bar{P}_{\mathrm{PV}}} g\left(P_{\mathrm{PV},i,j,h}\right)} + \mathrm{g} * \rho_{i,h} \tag{33}$$

$$\rho_{i,h} = \left|P_{\mathrm{PV},i,h,iter} - P_{\mathrm{PV},i,h,iter-1}\right| \tag{34}$$

$$\begin{cases} max\ (32), (33) \\ s.t. (9-12), (15), (18) \end{cases}, \tag{35}$$

where $g(P_{\mathrm{PV},i,j,h})$ is the continuous probability distribution function of prosumer *j*'s PV uncertainties that reflect on prosumer *i*, *iter* is the number of iterations in each game stage, ρ is the Euclidean distance between each PV generation scenario, g is the parameter of this distance.

Through these processes, the high dimension problem in each game stage changes to a decision model with continuous variables. The influence of the large size of state set on the proposed method's performance has been reduced to a small degree, only affects the range of the variables. Then the proposed game model can be solved by the general optimization methods.

Algorithm 1: Distributed solution algorithm for game G.

1. Setting the parameters $e_{i,h}$, $f_{i,h}$, $a_{i,h}$, $b_{i,h}$, $\alpha_{i,h}$, $c_{s,h}$, $c_{b,h}$ of the voltage regulation model, and the initial value $P_{\mathrm{load},i,h}$, $P_{\mathrm{PV},i,h}$, $Q_{0,i,h}$ of each prosumer.

For time slot $h = 1$

2. DSO calculates the power flow and getting the reactive power-voltage sensitivity matrix $\boldsymbol{S}_{Q,h}$ and active power-voltage sensitivity matrix $\boldsymbol{S}_{P,h}$, then calculate the initial node voltage $V_{0,i,h}$.
3. DSO calculate the weighting degree $d_{i,h}$ of each node.

For iteration iter $= 1$

For prosumer $i = 1$

4. Prosumers set their reactive power strategies based on (3-18) and get the node voltage through (8), (17), and active power curtailment $\Delta P_{\mathrm{PV},i,h}$ through (16):

$$u_{1,i,h}/u_{2,i,h} = \begin{cases} max\ U'_{1,i,h}/U'_{2,i,h}\left(\Delta Q_{i,h}, \Delta P_{\mathrm{PV},i,h}\right) \\ s.t., (9-12)\,,\ (15)\,, (18) \end{cases}$$

5. Each prosumer transmit the optimal strategies $A^*_{i,h}$, active power adjustment $\Delta P^*_{\mathrm{PV},i,h}$, and the PV uncertainties $g(P_{\mathrm{PV},i,h})$ to other prosumers for their own optimization combined with the $\boldsymbol{S}_{Q,h}$ and $\boldsymbol{S}_{P,h}$.

If $i = N$

End for prosumer

If $\max\limits_{i} |A^*_{i,h,iter} - A^*_{i,h,iter-1}| \leq 1e-3$

End for iteration

6. DSO calculates the power flow based on the prosumers' optimal strategies $\{A^*_{i,h} | i = 1 : N\}$, acquiring the optimal node voltage distribution.
7. Passing the prosumers' optimal strategies to the next game stage (time slot), then turn to step 2.

If $h = H$

End for

B. Distributed Solution Algorithm for the Stochastic Game

The proposed stochastic game model can be transferred to a mathematically solvable problem through the MDP and continuing discrete PV uncertainty. The interior-point optimization method uses to solve the optimization problem (i.e., voltage regulation model) of each prosumer. The optimization conducts in the UEMS of each prosumer, which is a distributed way with information transmission between them. During the solving process, when one prosumer gets its optimal results, its strategies (i.e., reactive power adjustment) with corresponding PV probability will transmit to other prosumers for their interaction, and the active power adjustment, network power flow, and node voltage will dynamically calculate by DSO based on the prosumers' strategies. Therefore, the distributed iterative solution method is adopted to get the NE based on the information transmission between each prosumer. The detailed process is shown as follows:

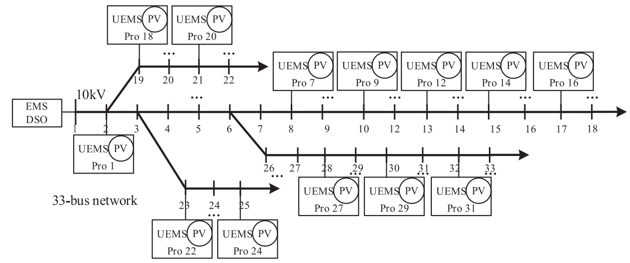

Fig. 4. The structure of voltage regulation system.

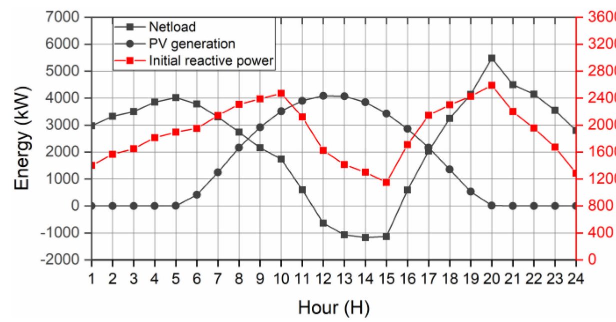

Fig. 5. The initial value of netload, PV generation, and reactive power.

TABLE I
PV INVERTER CAPACITY

PV inverter capacity (kVA)	Node
100	1, 2, 3, 4, 5, 10, 12, 14, 18, 20, 21, 25, 26
120	32, 11, 13, 15, 27, 28, 30
180	22
250	6, 7, 9, 16, 17, 19, 23, 29
300	8, 24, 31

V. CASE STUDY

A. Basic Data

The proposed voltage regulation method is verified by a test system (Fig. 4), which includes 32 prosumers with a PV inverter and a DSO to manage the system. The network parameters and structure come from the IEEE 33-node network, and the data of prosumers' load, PV generation, and inverter's capacity was taken from an industrial park in Guangdong Province, China. The load information, initial reactive power, and PV generation of all prosumers are shown in Fig. 5, the PV inverter capacity is listed in Table I. Because of the limited PV generation data, the PV uncertainties are hard to formulate as a continuous probability distribution function, a Gaussian distribution is used to build the PV distribution using the recorded PV data as the mean value. The parameters $a_{i,h}$, $b_{i,h}$, $e_{i,h}$, $f_{i,h}$, $r_{i,h}$ $c_{\mathrm{s},h}$, $c_{\mathrm{b},h}$, and $\alpha_{i,h}$ of voltage regulation model is set as 10, 1, 0.1, 0.1, 0, 0.4 CNY/kW, 1.0 CNY/kW, 20 CNY/kW.

B. Results of the Stochastic Game Model

1) Reactive and Active Power Strategies: Fig. 6 shows the voltage regulation of 32 prosumers in time slot 12 with high

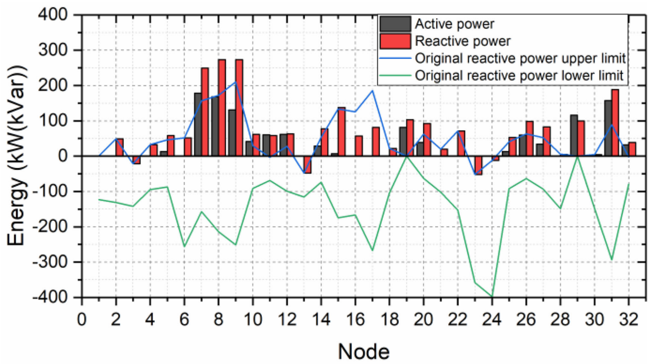

Fig. 6. The voltage regulation strategy in time slot 12.

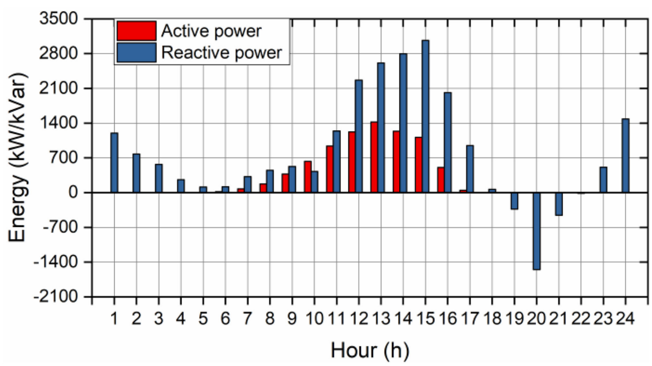

Fig. 7. All prosumers voltage regulation strategy in the entire time horizon.

PV generation. It is noted that the reactive power upper and lower limit is for the condition without PV curtailment. When the PV curtailment is considered, a more reactive power range will be allowed. Most of the prosumers absorb the reactive power because the high PV penetration led to high nodal voltage. Among them, the reactive power regulation of nodes (7-12, 14-15, 19-20, 25-27, 29, 31-32) exceeds the original upper limits of each PV inverter (as Eq. (9) shows), so that the active power should be cut down to provide a wider range for reactive power adjustment, even if some profit from PV generation will be reduced. From the network structure in Fig. 4, nodes (1-3, 19, 23) are close to the utility grid, the reactive power regulation is close to 0. The reason is that the capacity of the utility grid is much larger than the test system, the power flow change in the test system has little impact on the voltage of these nodes that close to the utility grid.

The overall voltage regulation strategies of 32 prosumers are shown in Fig. 7. From Fig. 7, the reactive power regulates a little in time slots 1-6, 18, 22, because the small load and PV generation have a slight influence on node voltage. In time slots 7-10, the amount of reactive power regulation is limited by the capacity constraints, however, there still require regulation to satisfy the voltage requirement, resulting in the active power curtailment. In time slots 11-17, a large amount of absorbing reactive power is decided to offset the out-of-limit voltage arisen from high PV penetration, which causes the short capacity of reactive power adjustment and the active power curtailment. In time slots 19-21, where the prosumers' load peak, the system faces the problem of voltage drop, and the PV inverter generates reactive power to support the node voltage.

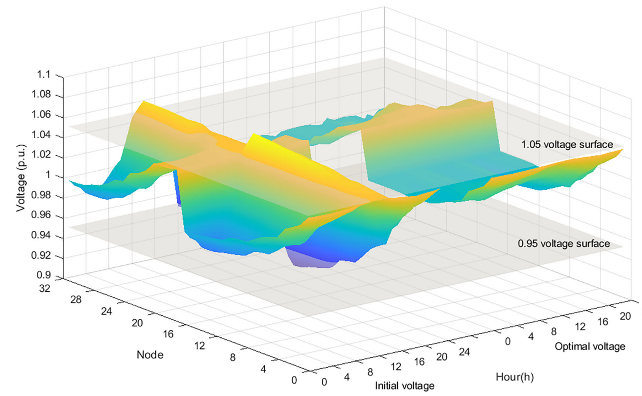

Fig. 8. The voltage results of each node in the entire time horizon.

2) Results of Node Voltage: Fig. 8 shows the voltage regulation results of 32 nodes (prosumers) in the entire time slots. From the perspective of nodes and voltage, the voltage of nodes (1-3, 19-23) can be maintained at 1.05 p.u. because they are close to the utility grid. The initial node voltage drops a little in some nodes far away from the utility grid due to the load consumption of prosumers, but it is still in the normal operation requirement. The PV generation satisfies the load demand while increasing the voltage of nodes far away from the utility grid (9-18), which exceeds the upper limit of normal operation requirement 1.05 p.u. Through the reactive and active power regulation, the voltage of these nodes that are directly affected by PV generation is reduced to the safe operation range, even close to 1 p.u. Therefore, the voltage regulation of the PV inverter can avoid the voltage drop arisen from heavy load consumption and voltage raise caused by high PV penetration.

From the perspective of time slots and voltage, it is clear that the out-of-limit node voltage (in time slots 11-15) is reduced to 1.05. p.u. to support the normal operation. The voltage drop resulted from a large amount of load consumption can also be increased to close to 1 p.u., especially in time slots 5 and 19-21, whose voltage is lower than the lower bound of voltage requirement 0.95 p.u. The average node voltage deviation (compared with 1 p.u.) is reduced from 3% to 1.5% through the optimization. Therefore, the node voltage of the whole network can be adjusted to the normal operating range in the entire time scale, which increases the voltage stability for system operation and reaches a high voltage quality for prosumers.

C. Comparison of Stochastic and Deterministic Results

To show the effectiveness of considering uncertain PV generation, the stochastic game results are compared with the deterministic results obtained by a non-cooperation game. The comparisons are conducted from the voltage deviation in the technical area and prosumers' utility in the economic area.

Fig. 9 shows the comparison results of node voltage obtained by the stochastic and deterministic methods. The comparison results are described by the difference of voltage deviation,

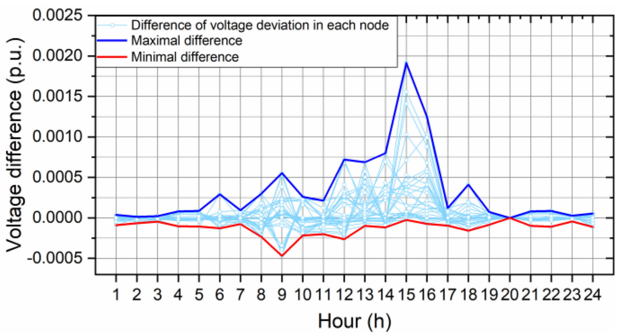

Fig. 9. The difference of stochastic and deterministic voltage deviation.

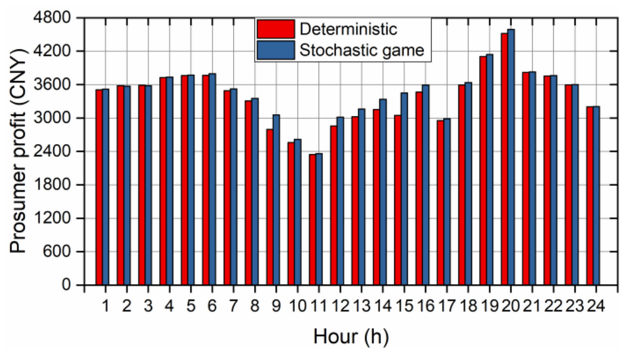

Fig. 10. The comparation results of voltage regulation utility.

which is calculated by the deterministic results minus stochastic results, positive values indicate that stochastic results are better than deterministic results, while negative values are the opposite. The negative value appears because the objective function of the stochastic game model is the utility function, which contains not only voltage deviation, but compensation and losses. It is clear that the stochastic game has a smaller voltage deviation in almost all nodes, and the maximal voltage deviation can 0.002 smaller than the deterministic results. The major difference between these two results is in the time slots 7-19 with the high PV generation, the reason is that the advantages of the stochastic game in terms of considering PV uncertainties.

The incremental utilities of all prosumers in 24 time slots are shown in Fig. 10, which is calculated by the difference between optimal utility and initial utility. The prosumers' optimal utility is got by the optimal strategies of all prosumers, while the initial utility is acquired by one of the prosumer's initial strategies and others' optimal strategies. The incremental utility of these two methods is higher in time slots 1-7, 12-16, and 19-23, and lower in time slots 8-11, 17-18, and 24. The reason is that the utility comes from the voltage deviation, from Fig. 8, it is clear that the time slots with large voltage deviation are the same to higher incremental utility, and small voltage deviation corresponds to the lower incremental utility.

In addition to the better voltage deviation results in the technical area, the prosumers' optimal utility obtained by the stochastic game method (83190CNY) is also higher than that of the deterministic method (81511CNY), which shows the better performance in the economy. Compared with the stochastic game method, the deterministic method has similar utility in time slots 1-6 and 21-24 because of the zero PV production.

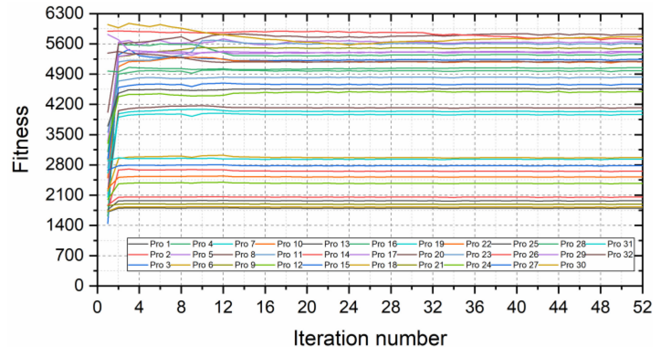

Fig. 11. The convergency curve of solution algorithm in time slot 12.

TABLE II
COMPUTATION TIME IN DISTRIBUTED AND CENTRALIZED METHOD

Prosumers number		33	52	72	85	118
Centralized computation time (s)		2.17	4.30	5.23	7.46	16.00
Distributed computation time (s)	All prosumers	50.42	90.59	127.60	165.48	215.73
	Each prosumer	1.57	1.61	1.79	1.97	1.84

In time slots 7-20, the utility of the stochastic game method is higher. That is because of the high PV production in these time slots, and the PV uncertainties and its occurrence probability are considered to get better results on large solution space.

D. Practical Feasibility, Computation Cost and Scalability

The proposed distributed solution algorithm is solved by a computer with Intel Core i7-8250 CPU 1.60 GHz, 16 G memory, and MATLAB 2018a was used as the testing environment. Only a few KB data require exchanged between each prosumer, which can be supported by the existing smart devices. The computation time of the proposed algorithm is 50.42s with 52 maximum iterations, each prosumer's convergency curve of time slot 12 is shown in Fig. 11.

To show the performance of distributed solution method, the computation complexity is compared with the centralized solution method in terms of the computation time, and the results are shown in Table II. It is clear that the computation time of the centralized method and distributed method increases with the prosumers' number. The reason is that the increased number of prosumers and network scales complicates the computation process. However, the distributed method implements in each prosumer's UEMS, which also increases with the prosumers' number, so that the computation time of each prosumer can be kept at a similar level.

To show the scalability of the proposed method, the IEEE 118-node network with 118 prosumers is also used as the test system. The voltage regulation results are shown in Fig. 12. It is clear that the voltage of all nodes can be maintained within 0.95 p.u. to 1.05 p.u., and much of node voltage close to 1 p.u., indicating the effectiveness of the proposed method in a large-scale system. Besides, from Table II, the computation time of the proposed method is 215.73s in the 118-node network, which

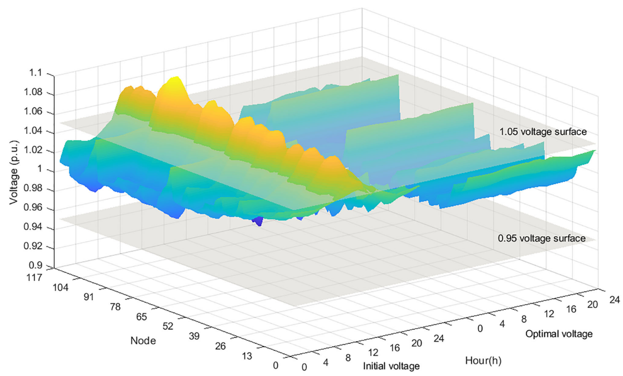

Fig. 12. Voltage regulation results in IEEE 118-node network.

is still within the second level and fully enough for the voltage regulation.

Moreover, because the voltage regulation method based on PV inverter and its reserved capacity can keep the voltage in the normal operation range, the reactive infrastructure investment (e.g., static var compensator) can be saved to increase the economy of the power grid.

VI. Conclusion

In this paper, we propose a stochastic game approach for distributed voltage regulation based on autonomous prosumers with PV uncertainties, where the stochastic game model is built with a designed incentive-based voltage regulation model. We first show the stochastic game results of the prosumers for voltage regulation in terms of reactive power adjustment and PV curtailment. Through these two strategies, the node voltage is regulated within the normal operation requirement in 24 hours and acquire the high voltage quality. Then we show the comparison between the stochastic game method and the deterministic method, get the conclusion that the stochastic game method can consider the PV uncertainties and corresponding occurrence probabilities to get better voltage deviation results in the technical area and utility results in the economic area. The voltage deviation obtained by the stochastic game method is almost 1% lower than that of the deterministic method, and the prosumers' utility also increases 1.7%. The practical feasibility, convergency, and scalability are also analyzed. Future research will relate to the economic significance of distributed voltage regulation by prosumers' PV inverters in terms of market implementation and saving infrastructure investment.

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