Saturation Effects in Equitable Demand Response Tariff Design

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Abstract—**As the adoption of smart home management devices grows, residential consumers become increasingly responsive to electricity prices. However, this price responsiveness cannot continue indefinitely as prices increase. Modeling this saturation effect is crucial to prevent demand response from becoming too costly, especially for consumers. This paper proposes an optimization model to design price response events that ensure energy equity by considering the income status and response saturation effect of each consumer. The proposed method uses energy burden to measure energy equity and builds a piecewise linear model to express the response saturation effect of each consumer. We formulate the tariff design problem as a mixed integer non-linear optimization model, which achieves a demand reduction target while minimizing energy burden and sharing the economic expense proportionally. We use real-world datasets in a case study to obtain personalized electricity prices, energy consumption, and energy burden for each consumer. We find that personalized tariffs effectively reduce energy burdens. By comparing the results with and without saturation effects, we conclude that modeling saturation effects can reduce energy burdens in demand reduction response events.**

Index Terms—**Demand response; saturation effect; energy equity; tariff design; social characteristics**

I. INTRODUCTION

Peak electricity demand is expected to continue to grow due to electrification, adoptions of electric vehicles, and increasing occurrences of weather extremes [1]. Demand response (DR) is an effective method to incentivize consumers to adjust energy consumption by setting specific electricity tariffs, thereby reducing the system's peak demand. However, higher prices of electricity cannot reduce demand indefinitely. During the 2021 winter storm Uri, the price of electricity surged to $9,000/MWh in Texas. However, many consumers still kept lights and heat on and eventually received outrageous electricity bills of more than ten thousand [2]. Many reports and research also mention the saturation effect of the price response, indicating demand elasticity diminishes at high price extremes [3], [4].

The saturation effect is critical to model a customer's price elastics but is often neglected in previous studies due to the modeling difficulty. Previous research usually considers the consumers' price response as linear or quadratic functions [5], [6]: modeling linearly prices response indicated by the change of energy consumption [7]; designing quadratic function to express response behavior [8], [9]. Building optimization models also incorporated price responses on the consumer side [10], which generally uses a bi-level optimization structure. Some research modeled the price response considering the influence of price elastics and studied the influence of different kinds of price elastics [11]. These studies expressed the price elastics as constant values from empirical estimates [12], [13], or as Gaussian distributions to represent elasticity variations [14]. Yet, previous price response studies focused on economics, either to reduce consumer costs or increase utility profits.

Energy equity is another critical topic in tariff design to ensure fair access to electricity, a fundamental need in modern society [15], [16]. Previous tariff designs assume the same tariff for all consumers. The electricity bill brought by the same electricity tariff will have a disparate proportion in consumer perception due to the difference in socioeconomic factors such as income [17]. This has resulted in some consumers opting for power outages to save money, which reduces life quality and equitable access to energy [3]. Extant research formulates a bi-level framework to study the energy equity between prosumers and consumers in distributed energy transitional energy systems, introducing energy expenditure incidence to trade-off the prosumers' and consumers' costs [18]. Multi-objective optimization models considering economic, social, and environmental factors are also adopted to develop justice-cognizant tariffs [19]. However, equitable tariff designs are significantly influenced by consumers' saturation effect, which is determined by various social factors and response behaviors. Furthermore, consumers' response behaviors tend to vary over time based on their energy consumption profiles. These factors present new challenges for tariff design, as disregarding the saturation effect or inaccurately considering it will reduce the tariffs effectiveness for DR and make it difficult to maintain equity among consumers.

To address the aforementioned research gap, this paper proposes an optimization model to design DR tariffs while achieving energy equity considering consumer-specific saturation effects. The paper provides the following contributions:

1) We use energy burden to measure energy equity and model response saturation effects of each consumer based on a piecewise linear model. We thus formulate the tariff design problem as a mixed-integer non-linear program-

ming (MINLP) problem with the goal of achieving a demand reduction target while ensuring energy equity.

2) We divide high and low-income consumers according to their energy burden and build their tariff model to minimize the difference between the energy burden and the average value and share the cost equally, respectively.
3) Our case study employs real-world datasets to design individualized electricity tariffs for each consumer in demand reduction events according to their social characteristics to achieve energy equity and show the necessity to consider the saturation effect.

The remaining of the paper is organized as follows: Section II introduces the consumers' price response and tariff design model. Section III presents the dataset and computation results, and Section VI concludes the paper with a discussion on future directions.

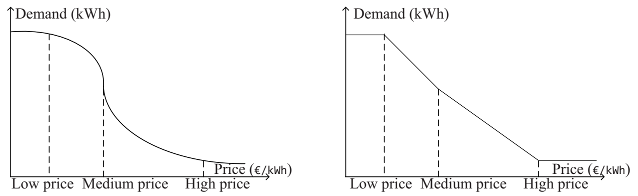
II. MODEL AND FORMULATION

We first present a piece-wise linear model for consumers' saturation effect, then present the tariff design optimization problem.

A. Consumers' individual price response with saturation effect

Each consumer will have individual price responses due to their unique income and electricity usage preferences and the operating time slots. Besides, consumers' price response has the saturation effect, i.e., when the price reaches a very high or very low level, load consumption changes a little or even keeps constant [4]. Under this effect, consumers' DR may differ from quadratic or linear prices response used by most studies.

Fig. 1 (a) [3], [4] shows consumers' price response with saturation effects. This curve can be linearized by multiple linear segments. The dynamic electricity tariff is designed as three types of prices during load peak, load valley, and normal load period. Therefore, consumers' price response can be linearized by four line segments, shown in Fig. 1 (b). The separating points are low prices in the peak load period, high prices in the load valley period, and normal prices in normal load periods, denoted by $\pi_{\text{low}}, \pi_{\text{high}}, \pi_{\text{med}}$, respectively. The linearized model is expressed as follows:



(a) Actual price response (b) Linearized prices response

Fig. 1. Consumer' prices response with saturation effect.

$$D_{i,t}(\pi_{i,t}) = \begin{cases} D_{i,t,\max}, & \text{if } 0 \le \pi_{i,t} \le \pi_{\text{low}}; \\ a_{i,t,1} * \pi_{i,t} + b_{i,t,1}, & \text{if } \pi_{\text{low}} \le \pi_{i,t} \le \pi_{\text{med}}; \\ a_{i,t,2} * \pi_{i,t} + b_{i,t,2}, & \text{if } \pi_{\text{med}} \le \pi_{i,t} \le \pi_{\text{high}}; \\ D_{i,t,\text{low}}, & \text{if } \pi_{\text{high}} \le \pi_{i,t} \end{cases} \tag{1}$$

where i is the index of the consumer, $i \in N$; t is the index of time slots, $t \in T$; $\pi_{i,t}$ is the electricity price of the consumer i in time slot t; $D_{i,t}(\pi_{i,t})$ is the load demand of consumer i in time slot t, which is the function of each consumer's electricity price; $a_{i,t,1}, a_{i,t,2}, b_{i,t,1}, b_{i,t,2}$ are parameters of the price response function in the second and third segments, which shows consumers' individual prices response are different for each other; $D_{i,t,\max}, D_{i,t,\min}$ are the individual energy demand when reaching saturation part of low prices and high prices.

We reformulate these piece-wise linear functions by introducing binary variables as the saturation curve is neither convex nor concave:

$$v_{i,t,1} * \pi_{\text{low}} \le \pi_{i,t,1} \le \pi_{\text{low}} \tag{2}$$

$$(\pi_{\text{med}} - \pi_{\text{low}}) * v_{i,t,2} \le \pi_{i,t,2} \le (\pi_{\text{med}} - \pi_{\text{low}}) * v_{i,t,1} \tag{3}$$

$$(\pi_{\text{high}} - \pi_{\text{med}}) * v_{i,t,3} \le \pi_{i,t,3} \le (\pi_{\text{high}} - \pi_{\text{med}}) * v_{i,t,2} \tag{4}$$

$$0 \le \pi_{i,t,4} \le G * v_{i,t,3} \tag{5}$$

$$v_{i,t,1} \ge v_{i,t,2}, v_{i,t,2} \ge v_{i,t,3} \tag{6}$$

$$\pi_{i,t} = \pi_{i,t,1} + \pi_{i,t,2} + \pi_{i,t,3} + \pi_{i,t,4} \tag{7}$$

$$D_{i,t} = D_{i,t,\max} - a_{i,t,1} * \pi_{i,t,2} - a_{i,t,2} * \pi_{i,t,3} \tag{8}$$

where G is a big value; $v_{i,t,1}, v_{i,t,2}, v_{i,t,3} \in \{0,1\}$ are binary auxiliary variables correspond to endpoint $\pi_{\text{low}}, \pi_{\text{med}}, \pi_{\text{high}}$; $\pi_{i,t,1}, \pi_{i,t,2}, \pi_{i,t,3}, \pi_{i,t,4}$ are electricity prices in each segment.

B. Formulation of tariff design problem

We propose the following model for the energy equity tariff design in demand reduction events. The overall goal is to reduce the lower-income consumers' energy burden while keeping the DSO's revenue. Therefore, high-income consumers should share some of the cost according to the DSO's revenue balancing requirements and take the risk of demand reduction to protect low-income consumers. Decision variables in the optimization problem include tariff segments $\pi_{i,t,j}$ and auxiliary binary variables $v_{i,t,j}$, where $j \in \{1, \ldots, 4\}$ is the segment index. The objective function is as follows:

$$\min \|E_{i,t} - E_{\text{ave},t}\|, \ \forall i \in N_{\text{low}} \tag{9}$$

$$E_{i,t} = \frac{D_{i,t}(\pi_{i,t}) * \pi_{i,t}}{I_i} \tag{10}$$

where $E_{i,t}$ is the energy burden of consumer i in time slot t; $E_{\text{ave},t}$ is the average energy burden of all consumers in time slot t, which is a constant; N_{low} is the set of low-income consumers, which is determined when energy burden less than the average value; I_i is the income of consumer i.

The energy burden reflects consumers' electricity bills as a percentage of their income. The larger the value, the higher the energy burden for the consumer. Low-income consumers tend to have a more significant energy burden, and sometimes they must cut off their energy usage to save money, reducing life quality [3]. Therefore, to realize energy equity, it is beneficial to reduce electricity prices for low-income consumers, either with subsidies or refunds, to reduce their energy burden. On the other hand, achieving energy equity does require higher-income consumers to pay more bills to support the savings of some lower-income consumers or to bear the load reduction during DR.

Thus, the problem includes the following constraints:

$$\text{s.t.}(2-6)$$

$$\sum_{i\in N} D_{i,t} \leq \alpha_t * \sum_{i\in N} D_{i,t,0}, \forall t\in T \tag{11}$$

$$E_{i,t} \leq \theta * E_{\text{ini},t} \tag{12}$$

$$|(\sum_{i\in N} \pi_{i,t} * D_{i,t} - P_{\text{ini},t})| \leq 0.5 * P_{\text{ini},t}, \forall t\in T \tag{13}$$

$$E_{i,t} \geq 0, \pi_{i,t} \geq 0, D_{i,t} \geq 0 \tag{14}$$

where α_t is the demand reduction ratio in time slot t; $P_{\text{ini},t}$ means the baseline profits, which is determined by the day-ahead market electricity prices; θ is the price cap of all consumers, which is reflected by energy burden; $D_{i,t,0}$ is the consumers' demand with baseline electricity prices.

(11) shows the demand reductions in the DR events; (13) shows that the operators' profit from using the new electricity tariff should within a range of baseline profit; (12) indicates the consumers' price cap, which means consumers cannot accepts excessively high electricity prices according to their income levels, and the price cap is reflected by the energy burden. (14) shows the non-negative constraints of decision variables, including prices, energy burden, and demand.

III. CASE STUDY

A. *Datasets*

In the case study, we use two datasets to derive the saturation effects of energy equity tariff designs in demand reduction events. One is from Low Carbon London (LCL) project [20], which includes a real DR dataset, providing DR profiles of 1100 consumers receiving time of use (ToU) tariffs and 4400 consumers receiving non-ToU flat rate at 30-minute granularity during the year 2013. The other is Commission for Energy Regulation (CER) Smart Metering Project [21], which includes over 5000 Irish consumers' energy consumption data and social survey data from 2009 and 2010. The energy consumption is the 30-minute granularity, and social survey data includes their income, family member, acceptable of electricity prices, etc.

In the LCL project, consumers are separated into three categories according to their income, named affluent, comfortable, and adversity. Besides, three different ToU tariffs are designed, which are high (67.20p/kWh), low (3.99p/kWh), and normal (11.76p/kWh). Consumers can adjust their energy consumption according to the tariffs. Using support vector machine (SVM) to get the baseline demand, denoted by $D_{0,i,t}$, which reflects the original energy consumption without ToU tariffs.

With the baseline energy consumption and DR, the price response behavior parameters used in (1) can be obtained as follows:

$$a_{i,t,1} = \Delta D_{i,t} / (\pi_{\text{med}} - \pi_{\text{low}}) \tag{15}$$

$$a_{i,t,2} = \Delta D_{i,t} / (\pi_{\text{high}} - \pi_{\text{med}}) \tag{16}$$

$$\Delta D_{i,t} = D_{i,t} - D_{0,i,t} \tag{17}$$

where $\pi_{\text{low}}, \pi_{\text{med}}, \pi_{\text{high}}$ corresponding to the low, normal, and high ToU tariffs in LCL projects.

We select 100 consumers from the CER smart meter project as the research object, obtain their daily energy consumption data and income information and classify them into three categories according to the setting of LCL project, and different categories have different price response behaviors. Then, using consumers' price response behavior from the LCL project reflects their DR preference. Note that consumers of the two projects have different energy consumption levels and price responses. Thus, the price response data is mapped according to the energy consumption ratio of CER consumers and LCL consumers, and the unique electricity price response of CER consumers can be obtained. Income is also mapped to each time slot based on energy consumption per time slot and used to calculate energy burden to differentiate low- and high-income consumers, as shown in Table I. Note that consumers' numbers are listed with their income level; some consumers have a relatively high income but still belong to low-income consumers due to their high energy consumption.

TABLE I
HIGH-INCOME AND LOW-INCOME CONSUMERS

High-income	2 5 9 10 12 14 19 20 21 22 24 25 26 31 34 38 39 41 43 44 47 48 49 50 52 59 60 66 67 68 69 72 74 76 78 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 96 97 98 99 100
Low-income	1 3 4 6 7 8 11 13 15 16 17 18 23 27 28 29 30 32 33 35 36 37 40 42 45 46 51 53 54 55 56 57 58 61 62 63 64 65 70 71 73 75 77 79 95

B. *Demand response results with saturation effect*

The case study considers a tariff design problem for a specific DR event following the LCL project setting. Instead of applying the same price to all households, we now consider the utility provides individualized DR rates to all participants based on their historical response data and income status to achieve a targeted demand reduction (2%) while ensuring energy equity, i.e., reduce demand by raising the price while also making sure the high price does not pose an extra energy burden to low-income consumers. The overall problem is an MINLP model with quadratic and bi-linear objective and constraints, and we solve the model using GUROBI 9.5.2 in the YALMIP toolbox of MATLAB 2022b, which supports the optimization of bi-linear terms [22]. The solving algorithm is implemented using a computer with i5-8250 CPU 1.60 GHz, 16 G memory. The computation time for 100 users in a time slot is about 5s.

The individualized electricity tariffs for energy equity are shown in Fig. 2. The tariffs reflect consumers' actual energy cost because consumers' true price response is saturation. Specifically, time-varying tariffs increase in high-income consumers to reduce energy consumption, and restricted by the price cap. Some low-income consumers prices also increase, but the they are secondary choice due to the price cap of high-income consumers.

Fig. 3 shows consumers' energy consumption reduction based on the response tariffs. Because of demand reduction events, consumers' load consumption should reduce. It is obvious to see that the demand of high-income consumers reduce more, and low-income consumers' demand keep the same or slightly reduce, which shows the effectiveness of our approach in achieving energy equity during demand reduction events. Most of the demand reduction are taken by the high-income consumers with higher electricity prices. The demand reduction of high-income consumer is also affected by their baseline demand, e.g., consumer 90 reduce the most demand (0.057) due to its higher baseline demand (2.163).

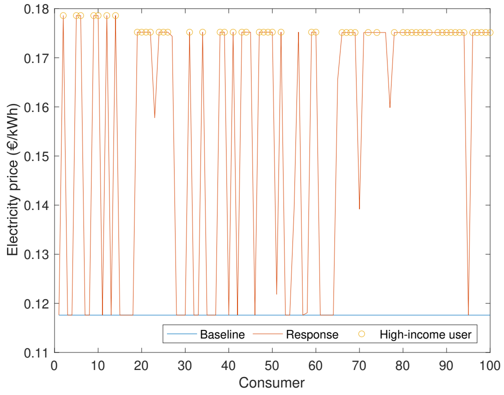


Fig. 2. Tariffs for all consumers.

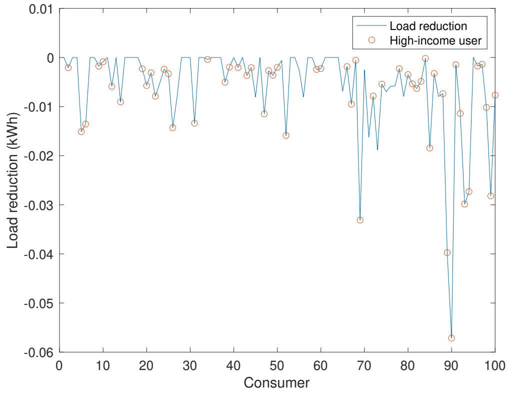


Fig. 3. Energy consumption reduction for all consumers.

Fig. 4 shows the energy burden of all consumers. Low-income consumers cannot reduce their energy burden by excessively increasing their energy consumption in response to prices. The reason is that operators' profit cannot increase or reduce their profit too much, consumers prices are restricted by the cap of, and the whole system should reduce demand. It is obvious that most demand reductions occur to high-income consumers, proportionally increasing the energy burden of most high-income consumers to the average of 6%. We also design a "One price" scenario with all consumers receiving the same tariff of 15.85p/kWh. The comparison results show that energy equity tariff design can reduce the energy burden for low-income consumers, and transfer these energy burden to high-income consumers to some extant, which is a fair way to reply to the demand reduction event.

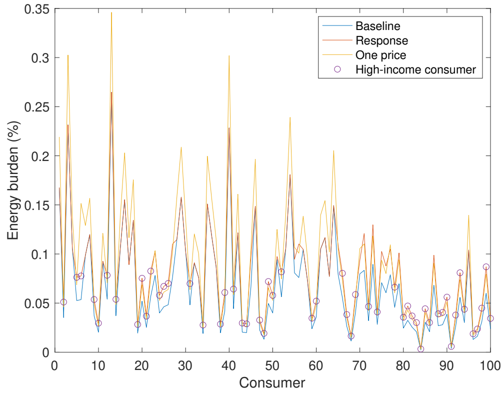


Fig. 4. Energy burden results for all consumers.

C. Comparison without saturation effect

This section compares the electricity tariff with and without the saturation effect. The latter formulates a linear model without the first and fourth segments of Eq. (1), and Fig. 5 shows the comparison results. Compared with the saturation scenarios, the linear scenario produces high tariffs for most high-income consumers. The reason is that energy consumption continues to decrease as tariff increase, and operators should set high electricity tariffs to maintain profits. As the figure shows, many of the tariffs exceed the price cap, which is unrealistic. Reducing these tariffs to the cap will result in profit losses for operators or less energy equity for consumers.

Moreover, since the saturation scenario reflects the actual price response curve, operators will profit more if they set tariffs without considering the saturation effect. The difference in consumers' energy cost shows in Fig.6 (calculated by Linear scenario minus Saturation scenario). Cost savings are lower for lower-income consumers because their tariff changes are smaller, meaning they do not receive high tariffs, which means they don't receive high tariff and are protected during the demand reduction event. However, most high-income consumers will pay more bills without the saturation influence, which is unfair to them. With enough income, these consumers may invest in photovoltaic devices and leave the DR program, bringing shock to the system. Thus, Considering the saturation effect in the design of energy equity tariffs can prevent operators from setting high and outrageous tariffs for high-income consumers to make enough profits, thereby reducing high-income consumers' expenditures.

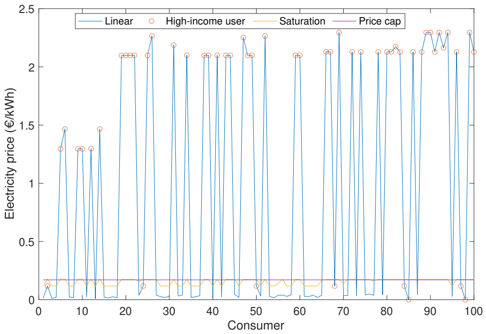
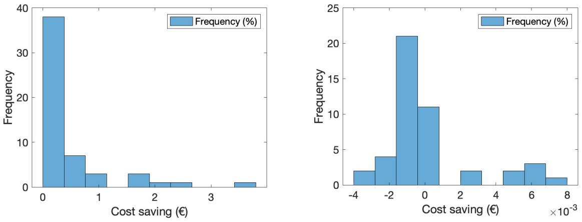


Fig. 5. Comparison of electricity prices.



(a) High-income consumers (b) Low-income consumers

Fig. 6. Consumers' cost saving in Saturation and Linear scenarios.

IV. CONCLUSION

This paper proposes a tariff design method for demand reduction events while achieving energy equity, considering the saturation effects of each consumer. Based on a dataset from a practical DR project, we obtain the personal price response for each consumer in different time slots and use it in another real dataset to calculate the energy equity tariff and corresponding energy consumption. The results show that the tariffs can reduce the total energy burden by 13.7% compared with the same high prices charged for all consumers during a demand reduction event. Consumers respond to demand reduction events in an equitable manner, with higher-income consumers' energy burdens increasing proportionally to their income levels, while some lower-income consumers are protected from the demand reduction event. We also benchmark with scenarios without considering saturation effects, and the results demonstrate that operators can set unreasonably high prices for most high-income consumers, significantly increasing high-income consumers' energy costs, which may discourage them from participating in future DR programs. The main constraint with the proposed approach is the computation complexity as it uses MINLP, while future research will investigate alternative methods more efficiently optimize tariffs for a large number of participants. Our future works also plan to consider response uncertainties in the tariff design.

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