A Prudent Framework for Understanding Risk-Awareness in Demand Response

-------Group meeting

Original title: Prudent Price-Responsive Demands

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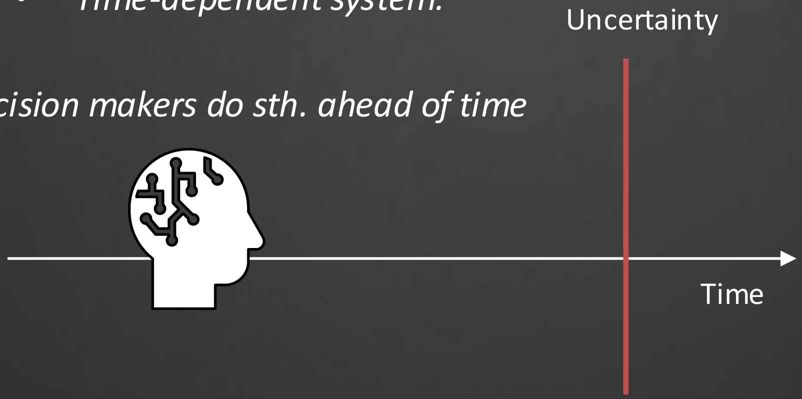
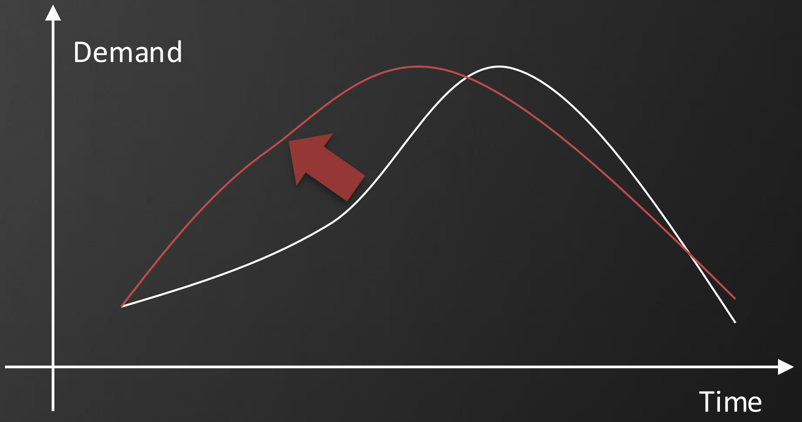
Prudent Price-Responsive Demands

- *Economic: seeing ahead, sagacity*
- *If uncertain events happen, prudent decision-makers will do sth. to respond to the event*
- *Time-dependent system:*

Decision makers do sth. ahead of time





Toy example:

Suppose you have a battery and participate in the real-time market with price uncertainty.

You need to decide on charge or discharge starting now until the price is realized

Now, I told you future price variance increase, but the expectation is the same.

What will you do?

- *Background*
- *Problem formulation*
- *Main Results*
- *Case Study and Conclusion*

Background

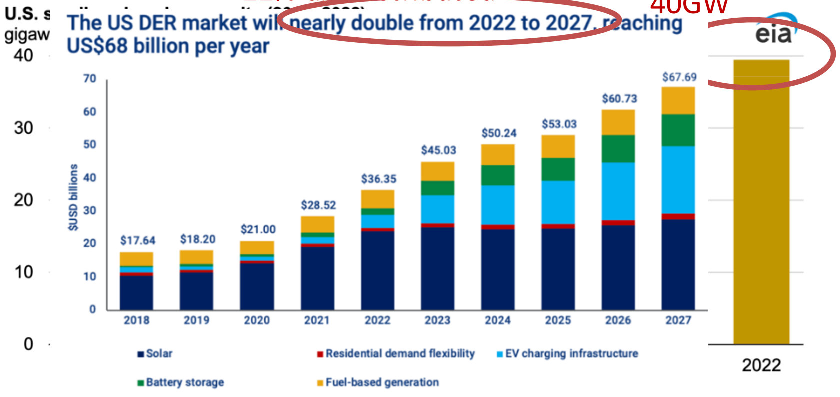
Motivation

- United States DER integration increase

179 GW solar capacity until 2023

22% are distributed

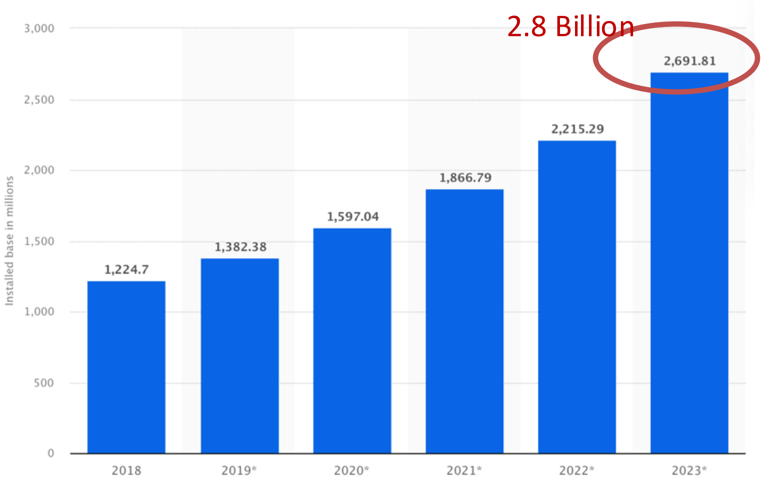
40GW



https://en.wikipedia.org/wiki/Solar_power_in_the_United_States
https://www.eia.gov/todayinenergy/detail.php?id=60341
https://www.woodmac.com/news/opinion/transformation-distributed-energy-resource-market/

- Consumers installed more smart home devices

2.8 Billion



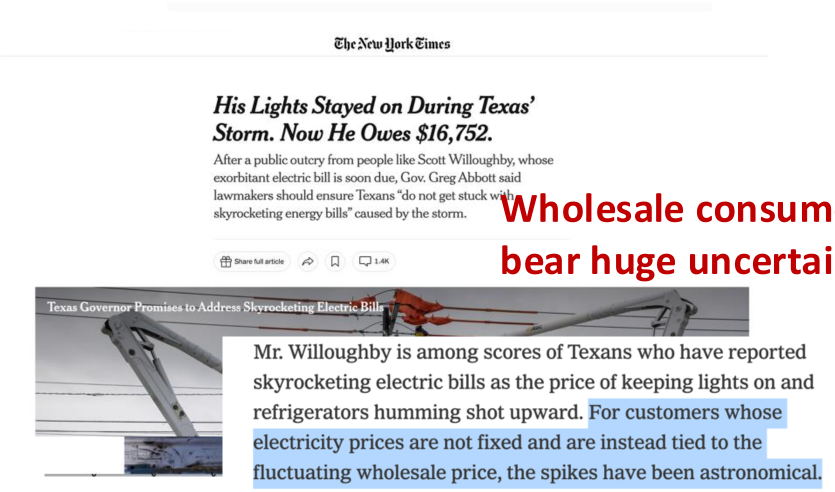
https://www.statista.com/statistics/1075749/united-states-installed-base-of-smart-home-systems/#statisticContainer
https://www.techtarget.com/iotagenda/definition/smart-home-or-building

Consumers become more responsive

Background

Dynamic prices incentivize consumers' responsiveness – uncertainty

- Wholesale markets are inherently uncertain.



The New York Times

His Lights Stayed on During Texas' Storm. Now He Owes $16,752.

After a public outcry from people like Scott Willoughby, whose exorbitant electric bill is soon due, Gov. Greg Abbott said lawmakers should ensure Texans "do not get stuck with skyrocketing energy bills" caused by the storm.

Texas Governor Promises to Address Skyrocketing Electric Bills

Mr. Willoughby is among scores of Texans who have reported skyrocketing electric bills as the price of keeping lights on and refrigerators humming shot upward. For customers whose electricity prices are not fixed and are instead tied to the fluctuating wholesale price, the spikes have been astronomical.

Wholesale consumers bear huge uncertainty

https://en.wikipedia.org/wiki/Solar_power_in_the_United_States
https://www.eia.gov/todayinenergy/detail.php?id=60341
https://www.woodmac.com/news/opinion/transformation-distributed-energy-resource-market/

- Utility companies adopt dynamic tariffs to incentivize demand responses.

Ameren Power Smart Pricing in Illinois

Hourly Prices for April 21, 2024

Retail consumer bear huge uncertainty

https://www.coned.com/en/accounts-billing/your-bill/time-of-use
https://www.srpnet.com/price-plans/residential-electric/time-of-use
https://www.oge.com/wps/portal/ord/residential/pricing-options/smart-hours
https://www.ameren.com/illinois/account/customer-service/bill/power-smart-pricing/

Understand complex risk-aware behaviors facing (price) uncertainty

Background

Literature

- **Data-driven**

https://www.javatpoint.com/group-discussion

Infrastructure

Privacy

Less data-driven previous works

Face limited application problem

Consumers naturally have high-dimensional and non-linear behavior

- **Model-driven**: Adopt decision-making models with **utility functions** to represent consumers' decision-making process
  - Quadratic $ax^2 + bx + c$
  - Piecewise linear

$$e_t = \begin{cases} E & \text{if } \theta_t < v_t(E) \\ v_t^{-1}(\theta_t) & \text{if } v_t(E) \le \theta_t \le v_t(0) \\ 0 & \text{if } \theta_t > v_t(0) \end{cases}$$

  - Conditional value at risk (CVaR) or robust

$$\mathrm{CVaR}_\alpha(\boldsymbol{X}; z) = \min_{z \in \mathbb{R}} \{z + \frac{1}{1-\alpha} \mathbb{E}\{[\boldsymbol{X} - z]^+\}\}$$

Lack of first-principle understanding of risk-aversion motivations

Highlight the need for a more sophisticated utility function formulation

Background

What do we do

$$\min_{\mathbf{x},\mathbf{y}_\xi} \quad f^{\mathrm{F}}(\mathbf{x}) + \boxed{\mathcal{R}_{\text{cost}}\left[f^{\mathrm{S}}(\mathbf{x},\mathbf{y}_\xi,\xi)\right]}$$

$$\text{s.t.} \quad \boldsymbol{h}^{\mathrm{F}}(\mathbf{x}) = \mathbf{0}, \quad \mathbf{g}^{\mathrm{F}}(\mathbf{x}) \le \mathbf{0},$$

$$\Big\downarrow$$

Risk operator

$$\min_{\mathbf{x},\mathbf{y}_\xi} \quad f^{\mathrm{F}}(\mathbf{x}) + \max_{P\in\mathcal{A}} \boxed{\mathbb{E}_P\left[f^{\mathrm{S}}(\mathbf{x},\mathbf{y}_\xi,\xi)\right]}$$

Expectation

+ ~~Risk term~~

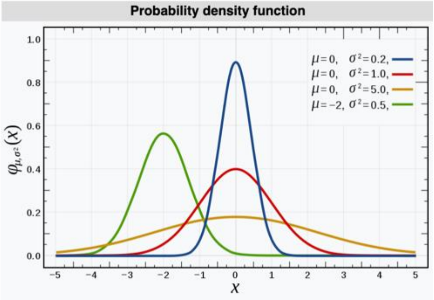
①

Intentionally

②

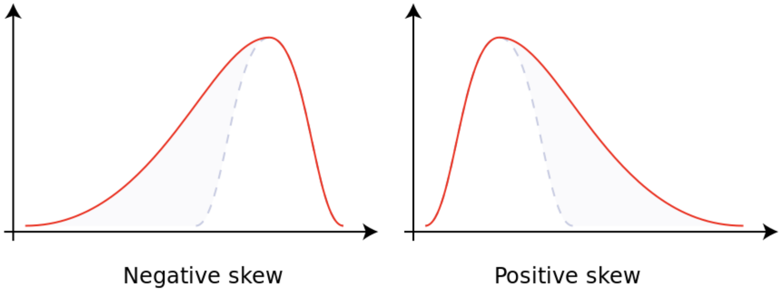
How does the future uncertainty distribution affect the risk-neutral decision-making process?

- Normal distribution – mean, variance



https://en.wikipedia.org/wiki/Normal_distribution

- Other practical distribution – mean, variance, **skewness,** etc. - shape

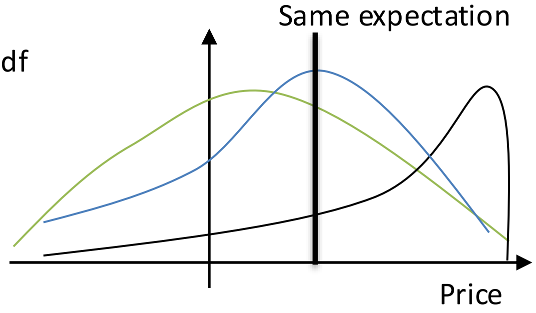


https://en.wikipedia.org/wiki/Skewness

Background

Contribution

- We establish a theoretical framework to model demand behavior to future volatile electricity prices We model demand with a risk-neutral cost-saving objective in a sequential decision-making context;



- We found that demand models with **quadratic cost functions** are **distribution-insensitive;**

- We prove that **super-quadratic cost functions** (higher order than two) result in **prudent demands**;

English: Current demand pattern affected by future uncertainty distribution

Math-wise: third-order derivative of the utility function

$$\frac{\partial^3 G_t(p_t)}{\partial p_t^3}$$

- We use a simulation with a real-world case to verify our results.

- *Background*
- *Problem formulation*
- *Main Results*
- *Case Study and Conclusion*

Problem Formulation

Demand model

Discrete time-varying system

Linear system transition

Risk-neutral, cost-saving objective

State cost

$$\min_{p_t} \mathbb{E}_{\Lambda_t} \sum_{t=1}^{T} \left[\lambda_t p_t + C_t(x_t) + G_t(p_t)\right] + V_T(x_T),$$

Energy cost

Action cost

End value function: for value continuity, set to 0

s.t.

$$x_t = Ax_{t-1} + p_t,$$

p_t is non-anticipatory

λ – uncertain price

P – power consumption (battery charging/discharging)

X – state (battery SOC)

***Cost function modeling soft and hard constraints**

Stochastic dynamic programming reformulation

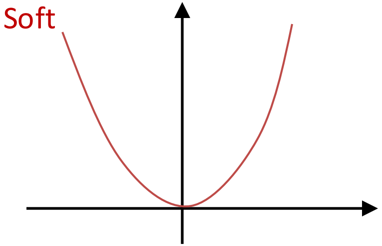
Working backward and recursively solving **a single-stage** optimization for all time *t*

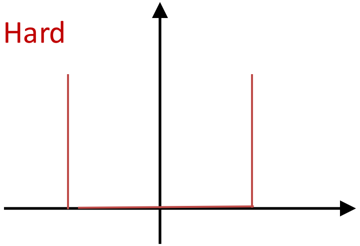
$$Q_{t-1}(x_{t-1}|\lambda_t) = \min_{p_t} \lambda_t p_t + C_t(x_t) + G_t(p_t) + V_t(x_t) \tag{3a}$$

$$V_t(x_t) = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})] \tag{3b}$$

$$\text{s.t. } x_t = Ax_{t-1} + p_t. \tag{3c}$$

Value function: rewards from the future about the current decision, it is a function of time-dependent state value.

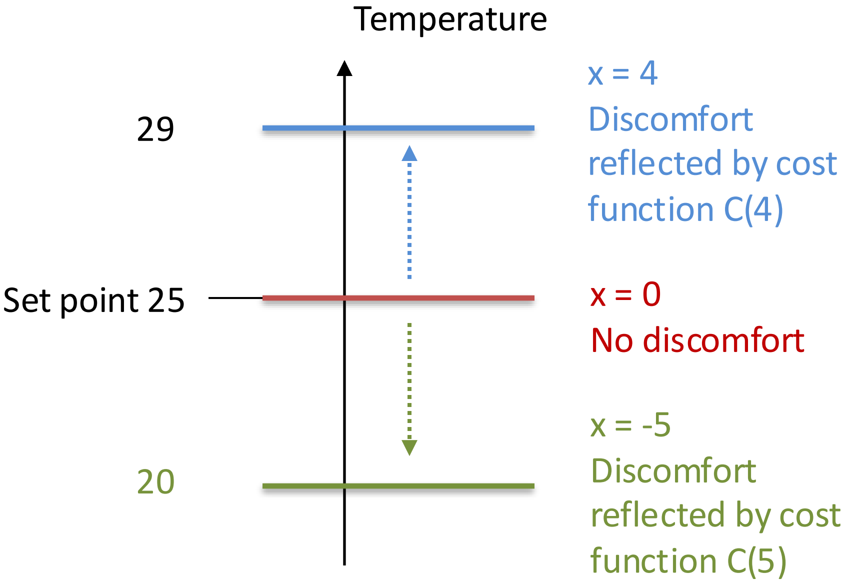




Problem Formulation

Definition - Normalized power and state cost

HVAC system (air conditioning)



Temperature

29

x = 4
Discomfort reflected by cost function C(4)

Set point 25

x = 0
No discomfort

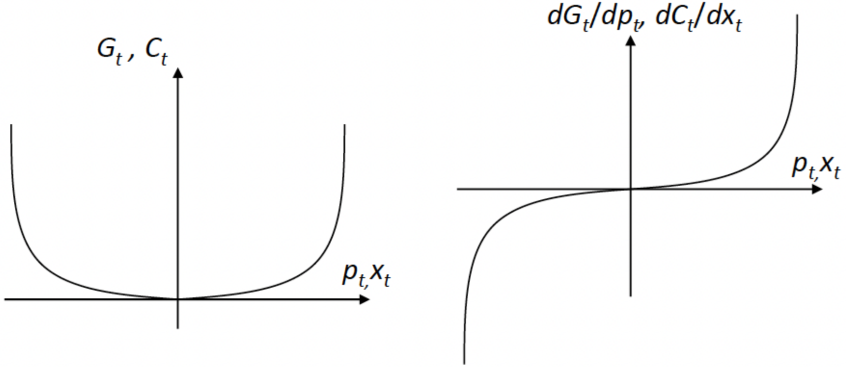
20

x = -5
Discomfort reflected by cost function C(5)

- The system is in equilibrium at zero power and state;
- Deviate from reference (0) increase discomfort (cost);
- Highlight our focus on disturbances and variations.

Definition/assumption

Convex and continuous



- *Background*
- *Problem formulation*
- *Main Results*
- *Case Study and Conclusion*

Main Results

Theorem 1 – Distribution-insensitive demand models

Demand model:

Quadratic action cost

$$Q_{t-1}(x_{t-1}|\lambda_t) = \min_{p_t} \lambda_t p_t + C_t(x_t) + G_t(p_t) + V_t(x_t)$$

Quadratic State cost

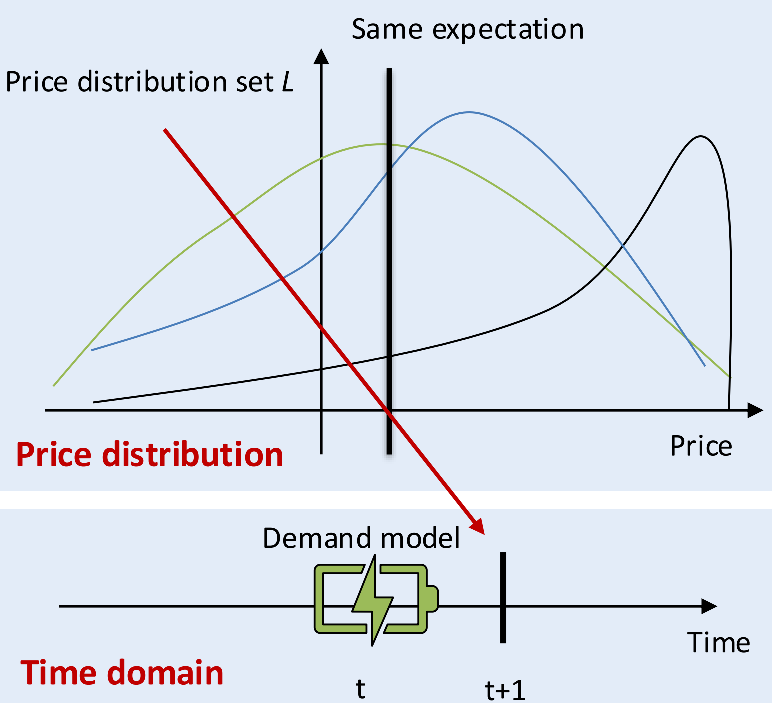
Set end value function to 0

$$V_t(x_t) = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})]$$

$$\text{s.t. } x_t = Ax_{t-1} + p_t.$$

Quadratic function: $G_t(p_t) = \frac{a_\mathrm{p} p_t^2}{2}, a_\mathrm{p} > 0,$

Demand model



Theorem - **The demand model at time t-1 is distribution-insensitive to price distribution at time t**

$$\mathbb{E}_{\Gamma_{t+1}}[Q_t(x_t|\lambda_{t+1})] = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})].$$

Value function with different uncertainty distribution

Main Results

Sketch of the proof:

Demand model

State-related cost $h = c + v$

$$Q_{t-1}(x_{t-1}|\lambda_t) = \min_{p_t} \lambda_t p_t + \boxed{C_t(x_t)} + G_t(p_t) + \boxed{V_t(x_t)} \tag{3a}$$

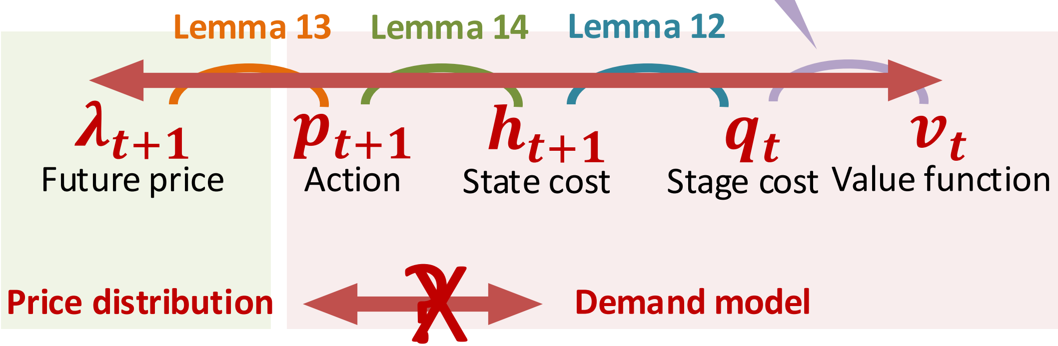
$$V_t(x_t) = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})] \tag{3b}$$

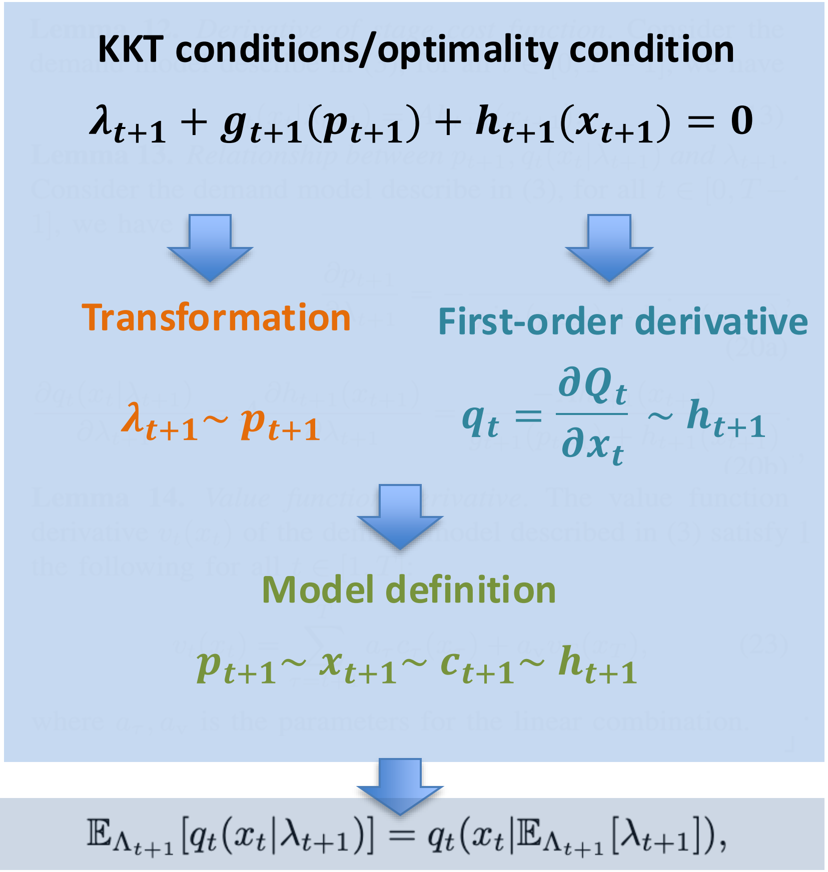
$$\text{s.t. } x_t = Ax_{t-1} + p_t. \tag{3c}$$

Capital letter: function

Lowercase letter: function derivative

$$c_t(x_t) = \partial C_t(x_t)/\partial x_t$$





KKT conditions/optimality condition

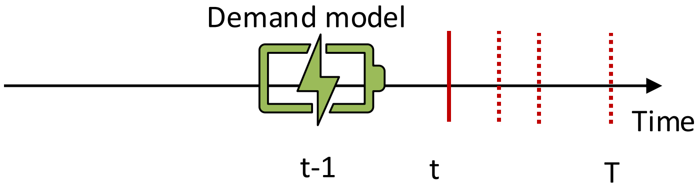
$$\lambda_{t+1} + g_{t+1}(p_{t+1}) + h_{t+1}(x_{t+1}) = 0$$

$$\mathbb{E}_{\Lambda_{t+1}}[q_t(x_t|\lambda_{t+1})] = q_t(x_t|\mathbb{E}_{\Lambda_{t+1}}[\lambda_{t+1}]),$$

Main Results

Corollary – time extrapolation

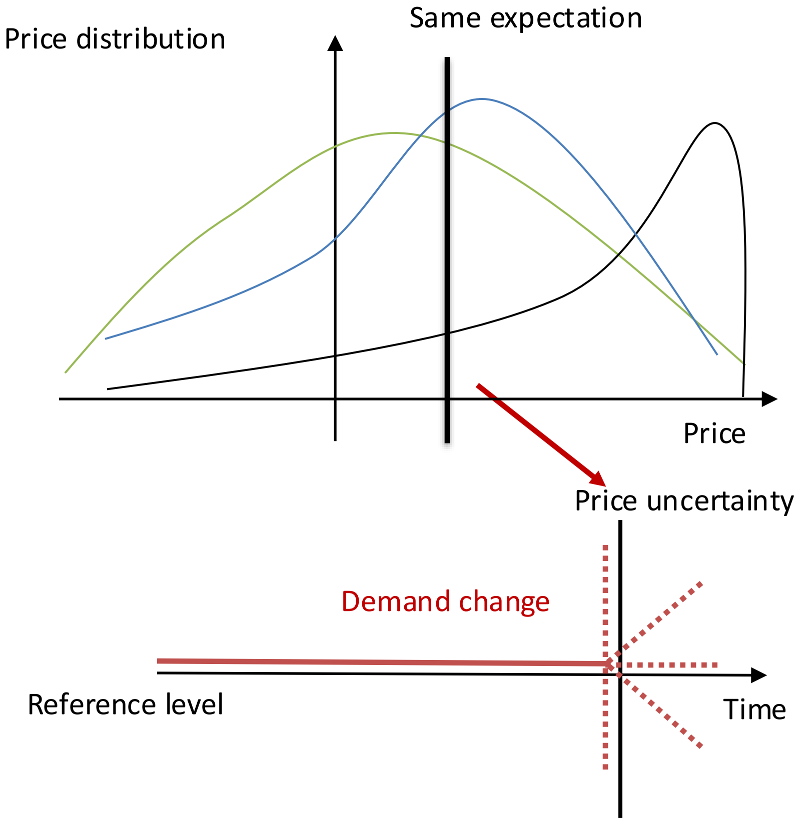
Theorem - The demand model at time t-1 is distribution-insensitive to price distribution at time t



Distribution insensitive to all future time

Key takeaway

- Demand with quadratic cost function is independent of the future price distribution but only the expectation;
- Widely used quadratic models fail to capture real risk-aware decision behaviors, as they inherently ignore distributional effects and may thus lead to unintended demand pattern changes



Main Results

Corollary – distribution sensitive demand

Demand model:

$$Q_{t-1}(x_{t-1}|\lambda_t) = \min_{p_t} \lambda_t p_t + C_t(x_t) + G_t(p_t) + V_t(x_t) \tag{3a}$$

$$V_t(x_t) = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})] \tag{3b}$$

$$\text{s.t. } x_t = Ax_{t-1} + p_t. \tag{3c}$$

Quadratic function:

$$G_t(p_t) = \frac{a_\mathrm{p} p_t^2}{2}, a_\mathrm{p} > 0,$$

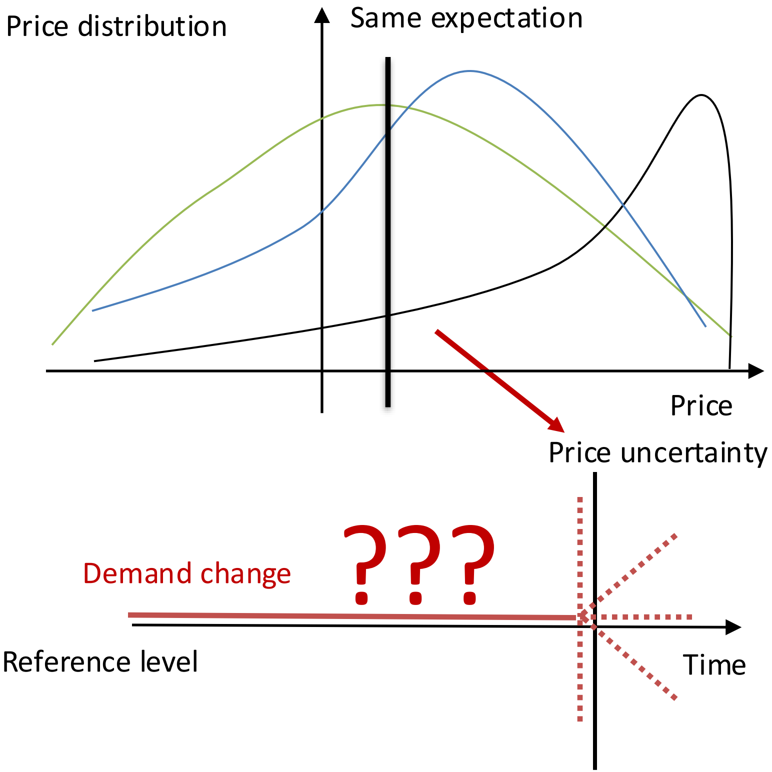
Super quadratic function:

$$\frac{\partial^3 G_t(p_t)}{\partial p_t^3} \neq 0.$$

Takeaway – problem

Practical situations challenges distribution-insensitive conditions:

- Devices show higher-order cost function performance (thermal comfort and hard constraints)
- Practical price distribution – not symmetrical with zero-mean



Motivated super quadratic formulation – prudent demand

Main Results

Theorem 2 – Prudent demand models

Demand model:

Super Quadratic State cost

Quadratic State cost

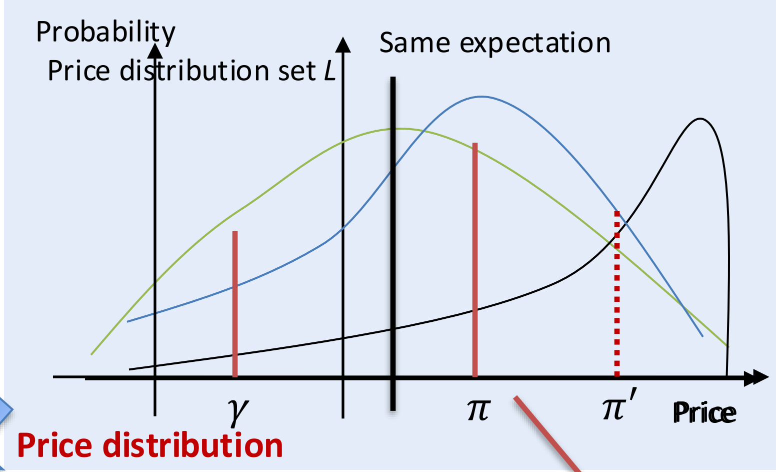
Set to 0

$$Q_{t-1}(x_{t-1}|\lambda_t) = \min_{p_t} \lambda_t p_t + C_t(x_t) + G_t(p_t) + V_t(x_t)$$

Quadratic action cost

$$V_t(x_t) = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})]$$
$$\text{s.t. } x_t = Ax_{t-1} + p_t.$$

Demand model



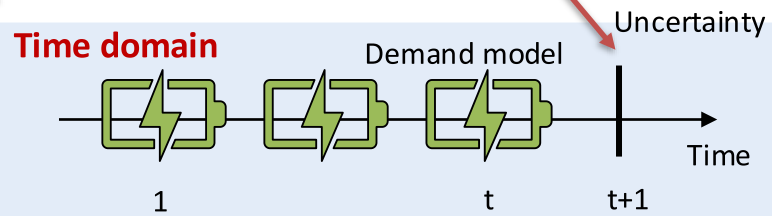
Price distribution

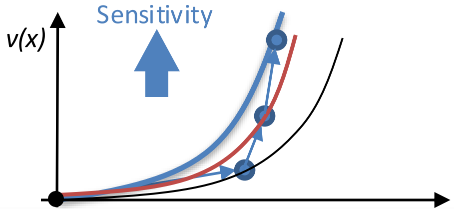
Theorem - **The demand model before time t satisfies prudent and its sensitivity condition to price distribution at time t+1**

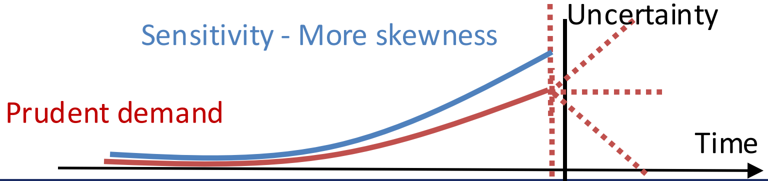
$$\boxed{\mathbb{E}_{\Gamma_{\tau+1}}}[Q_\tau(x_\tau|\lambda_{\tau+1})] > \boxed{\mathbb{E}_{\Lambda_{\tau+1}}}[Q_\tau(x_\tau|\lambda_{\tau+1})] > Q_\tau(x_\tau|\boxed{\mathbb{E}_{\Lambda_{\tau+1}}}[\lambda_{\tau+1}]) > 0, \forall \tau_0 < \tau \le t.$$

γ, π' γ, π

Value function with different uncertainty distribution







Main Results

Sketch of the proof:

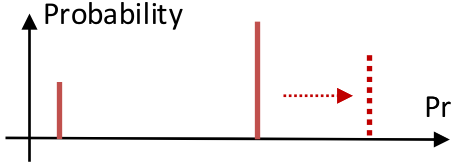
State-related cost $h = c + v$

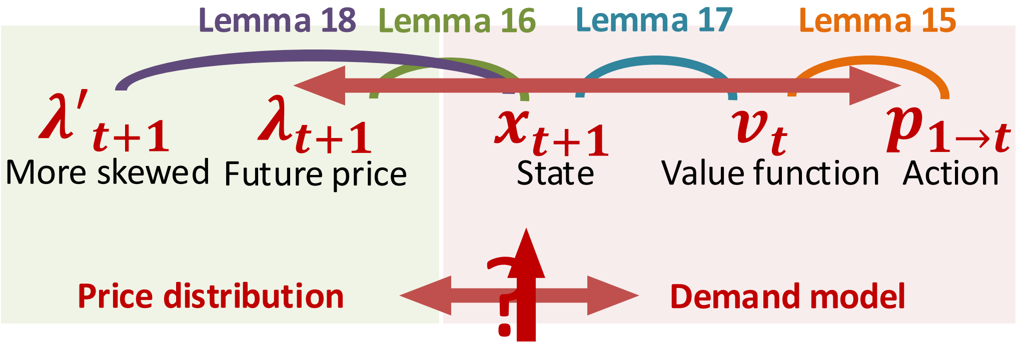
Demand model

$$Q_{t-1}(x_{t-1}|\lambda_t) = \min_{p_t} \lambda_t p_t + \boxed{C_t(x_t)} + G_t(p_t) + \boxed{V_t(x_t)} \quad (3a)$$

$$V_t(x_t) = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})] \quad (3b)$$

$$\text{s.t. } x_t = Ax_{t-1} + p_t. \quad (3c)$$





Lemma 18 Lemma 16 Lemma 17 Lemma 15

λ'_{t+1} More skewed

λ_{t+1} Future price

x_{t+1} State

v_t Value function

$p_{1\to t}$ Action

Price distribution ? Demand model

KKT conditions/optimality condition

$$p_1 + \sum_{\tau=1}^{t} c_\tau(x_\tau) + \overset{v_t}{\boxed{\mathbb{E}_\lambda[c_{t+1}(x_{t+1})]}} = 0$$

Function property

$x_{t+1} \sim \emptyset(\lambda_{t+1})$

Connect $v_t \sim \lambda_{t+1}$

Showing $v_t > 0$, then $x_t > 0$

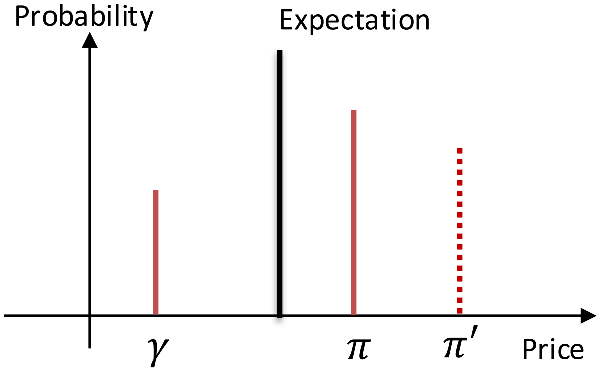
More skewed uncertainty

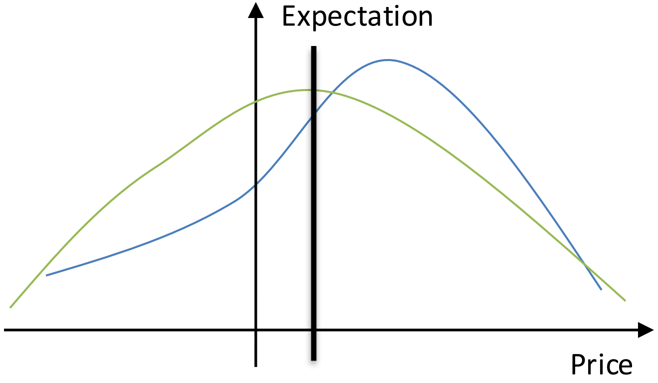
$\partial v_t / \partial \pi > 0$

$$\mathbb{E}_{\Gamma_{t+1}}[Q_t(x_t|\lambda_{t+1})] > \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})] > Q_t(x_t|\mathbb{E}_{\Lambda_{t+1}}[\lambda_{t+1}]) > 0.$$

Main Results

Corollary – Distribution & sensitivity extrapolation





Corollary – Strict condition

- Prudent theorem

$$\mathbb{E}_{\Gamma_{\tau+1}}[Q_\tau(x_\tau|\lambda_{\tau+1})] \geq \mathbb{E}_{\Lambda_{\tau+1}}[Q_\tau(x_\tau|\lambda_{\tau+1})] \geq 0, \forall \tau \leq t$$

- Demand model

$$Q_{t-1}(x_{t-1}|\lambda_t) = \min_{p_t} \lambda_t p_t + C_t(x_t) + G_t(p_t) + V_t(x_t) \quad \text{(3a)}$$

$$V_t(x_t) = \mathbb{E}_{\Lambda_{t+1}}[Q_t(x_t|\lambda_{t+1})] \quad \text{(3b)}$$

$$\text{s.t. } x_t = Ax_{t-1} + p_t. \quad A < 1 \quad \text{(3c)}$$

$$f_\Gamma \qquad \in \mathcal{X}_t,$$

$$x_{\tau_0} \approx A^{t-\tau_0} x_t \to 0$$

$$f_{\Gamma_t}(\pi_{\Gamma,t}) \geq f_{\Lambda_t}(\pi_{\Lambda,t}), \forall \pi_{\Gamma,t} > \pi_{\Lambda,t}, \{\pi_{\Gamma,t}, \pi_{\Lambda,t}\} \in \mathcal{X}_t,$$

- Strict condition $f_{\Gamma_t}(\pi_{\Gamma,t}) \leq f_{\Gamma_t}(\gamma_t), \forall \{\pi_{\Gamma,t}, \gamma_t\} \in \mathcal{X}_t,$

$$\mathbb{E}_{\Gamma_{\tau+1}}[Q_\tau(x_\tau|\lambda_{\tau+1})] > \mathbb{E}_{\Lambda_{\tau+1}}[Q_\tau(x_\tau|\lambda_{\tau+1})] > 0, \boxed{\forall \tau_0} < \tau \leq t.$$

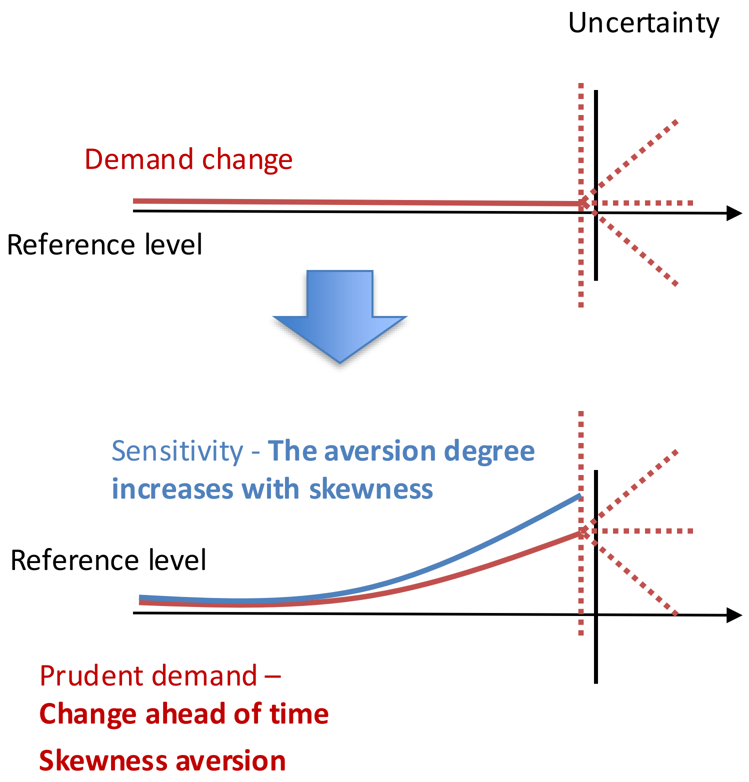
$$f_{\Gamma_t}(\pi_{\Gamma,t}) - f_{\Lambda_t}(\pi_{\Lambda,t}) > f_{\Lambda_t}(\gamma_t) - f_{\Gamma_t}(\gamma_t), \forall \{\pi_{\Gamma,t}, \pi_{\Lambda,t}\} \in \mathcal{X}_t.$$

Main Results

Key takeaway

- Prudent demand's value function increases with the future price distributional factors, with the same expectation;
- This preparatory behavior reflects an inherent risk-averse response in the demand model; we suggest that the commonly used risk-averse formulations are approximations of a more realistic, higher-order structure.
- The demand level change due to prudence scales proportionally with the skewness of the price distribution, showing the skewness aversion behavior
- Our results align with the prudence definition from economics

$$\frac{\partial^3 V_t}{\partial x_t^3} > 0$$



Outlier: Symmetrical distribution with the expectation of zero;

- *Background*
- *Problem formulation*
- *Main Results*
- *Case Study and Conclusion*

Case Study

Basic setting

- Quadratic action cost function: $G_t(p_t) = \frac{a_\mathrm{p} p_t^2}{2},$
- Log barrier state cost function:

$$C_t(x_t) = -\alpha_\mathrm{c} \ln(x_\mathrm{max} - x_t) - \alpha_\mathrm{c} \ln(x_\mathrm{max} + x_t) + 2\alpha_\mathrm{c} \ln x_\mathrm{max},$$

$$c_t(x_t) = \frac{\alpha_\mathrm{c}}{x_\mathrm{max} - x_t} - \frac{\alpha_\mathrm{c}}{x_\mathrm{max} + x_t}$$

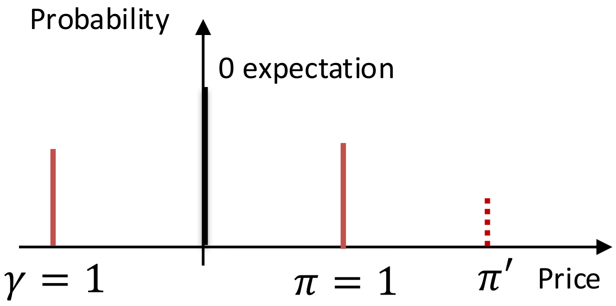
- Parameter:

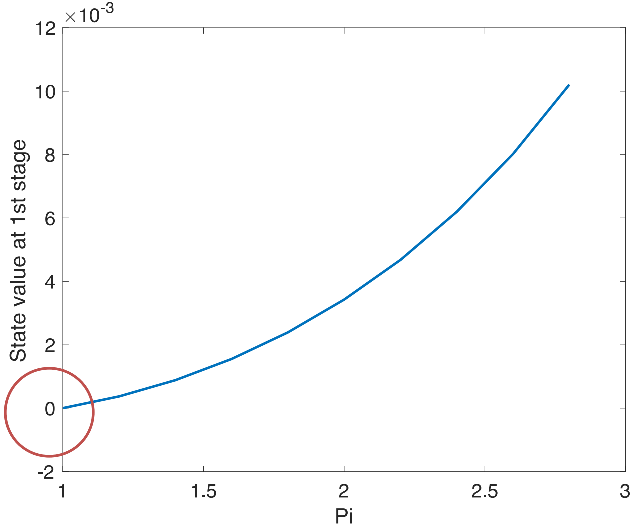
$$\alpha_\mathrm{c} = 0.5, A = 1, V_T = 0, a_\mathrm{p} = 1, x_\mathrm{max} = 20$$

An illustration example

- 2-stage, 2-point price distribution with 0 expectation

$$x_0 = 0, \gamma = -1, \pi = 1 \nearrow$$



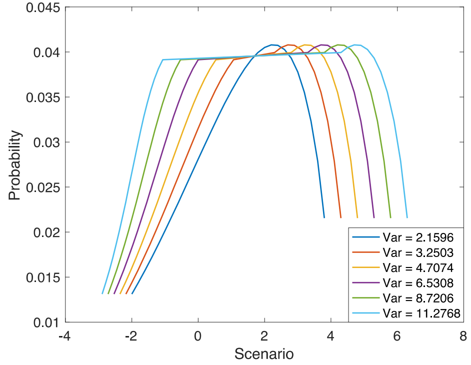


Symmetrical uncertainty with 0 expectation and 0 initial state

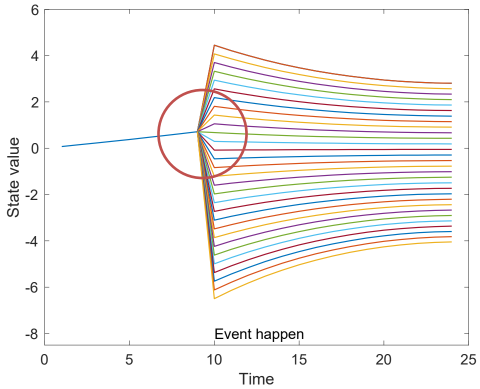
Case Study

Continuous prudent demand

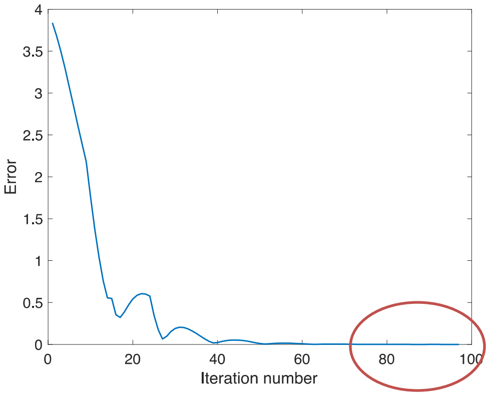
- 24 stages
- The event happens at the 10th stage
- 6 skewed price distributions with the same expectation and different variance (skewness)



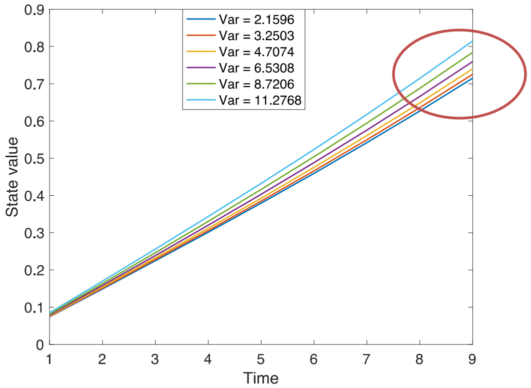
State under 1st price distribution:
Prudent demand increases before event happen



Convergence under 1st price distribution:
Calculation time: 2s.



State before 10th with all distributions:
skewness aversion

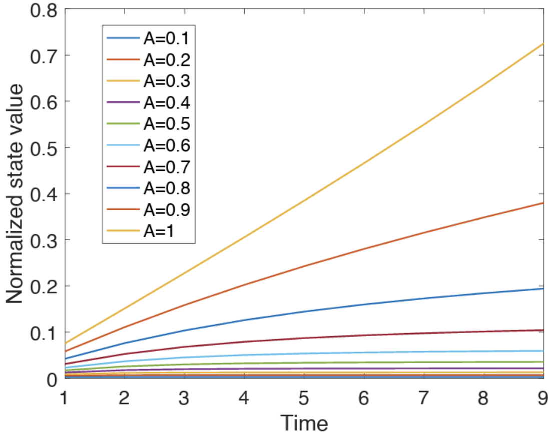


Case Study

Sensitivity of model parameters

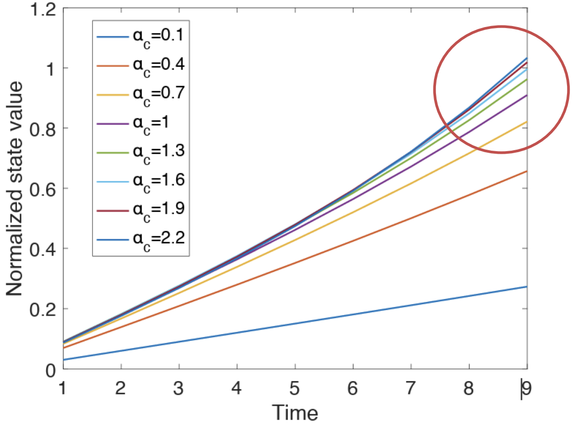
- 24 stages
- The event happens at the 10th stage
- 6 skewed price distributions with the same expectation and different variance (skewness)

State under 2nd price distribution:
Lower discount weaken the prudent influence



- Discount factors from 0.1 to 1 - A
- State cost function parameters - \alpha

Precautionary saving behavior persists greater potential loss from uncertainty at the event time for the same state value, inducing stronger precautionary saving behavior. But, with a saturation effect. The cost of precautionary saving begins to outweigh the cost of expected risk at the event time.



Case Study

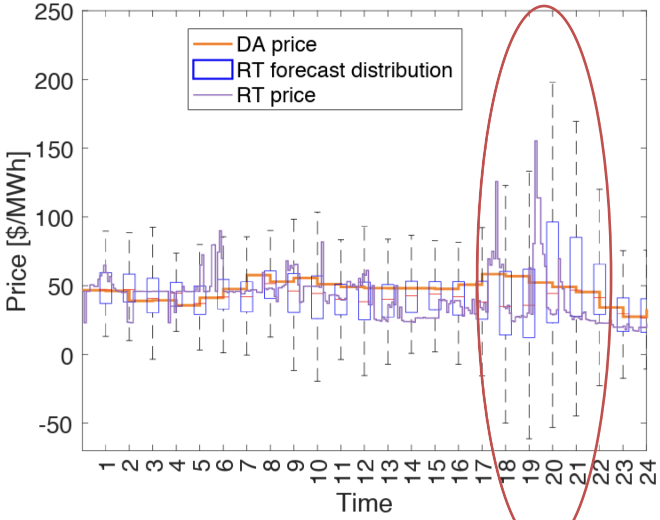
Real-world case study

- NYISO DA and RT price
- Battery doing arbicharge under non-anticipatory price uncertainty

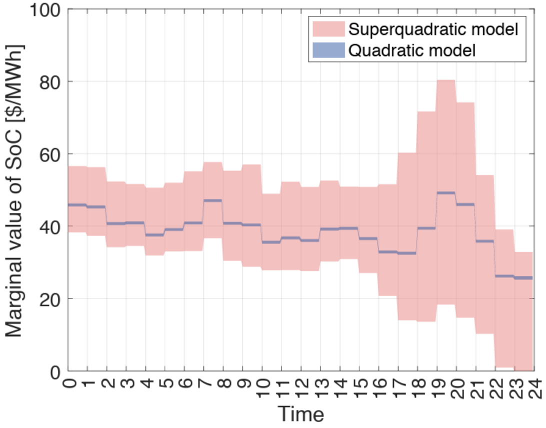
- Superquadratic model with hard constraints
- Quadratic model with quadratic penalty

- Superquadratic model exhibits greater variability
- Quadratic model captures the mean value of RT price uncertainty
- Superquadratic formulation provides richer and more responsive decision behavior.

20-21



(a) Price distribution from NYISO



(b) Battery SoC marginal value range under quadratic and superquadratic formulation

Interpretation and Conclusion

Conclusion

- We show that risk-aware behaviors in demand response originate from superquadratic state-dependent cost functions and price uncertainty with skewed distributions;
- We obtain such results through developing a novel theoretical demand response framework that combines non-anticipatory multi-stage decision-making with superquadratic cost functions;
- We introduce the concept of prudent demand, which is the first principle for risk-averse behavior despite a risk-neutral objective.
- Future price uncertainty affects immediate consumption decisions, and the extent of this response scales proportionally with the skewness of the price distribution

Interpretation and Conclusion

Practical implication

- Practitioners and policymakers should adopt more sophisticated demand models, either through higher-order utility function formulations or the adoption of more accurate risk terms, especially considering physical and behavioral response limitations, thus, capturing real demand behavior and better accounting for demand pattern changes to efficiently design time-varying tariffs
- Preparatory savings naturally provide additional backup capacity to the system ahead of emergencies. Thus, operators should not only focus on the event time itself, but also schedule additional generation or implement price incentives in advance to prevent unintended demand peaks.
- Discounting factors and the sensitivity of demand to state value changes are important for accurately quantifying prudent behavior.
- anticipating and preparing for tail-risk events and controlling the price ceiling exposed to customers, avoiding the direct exposure of retail consumers to highly volatile wholesale market prices.

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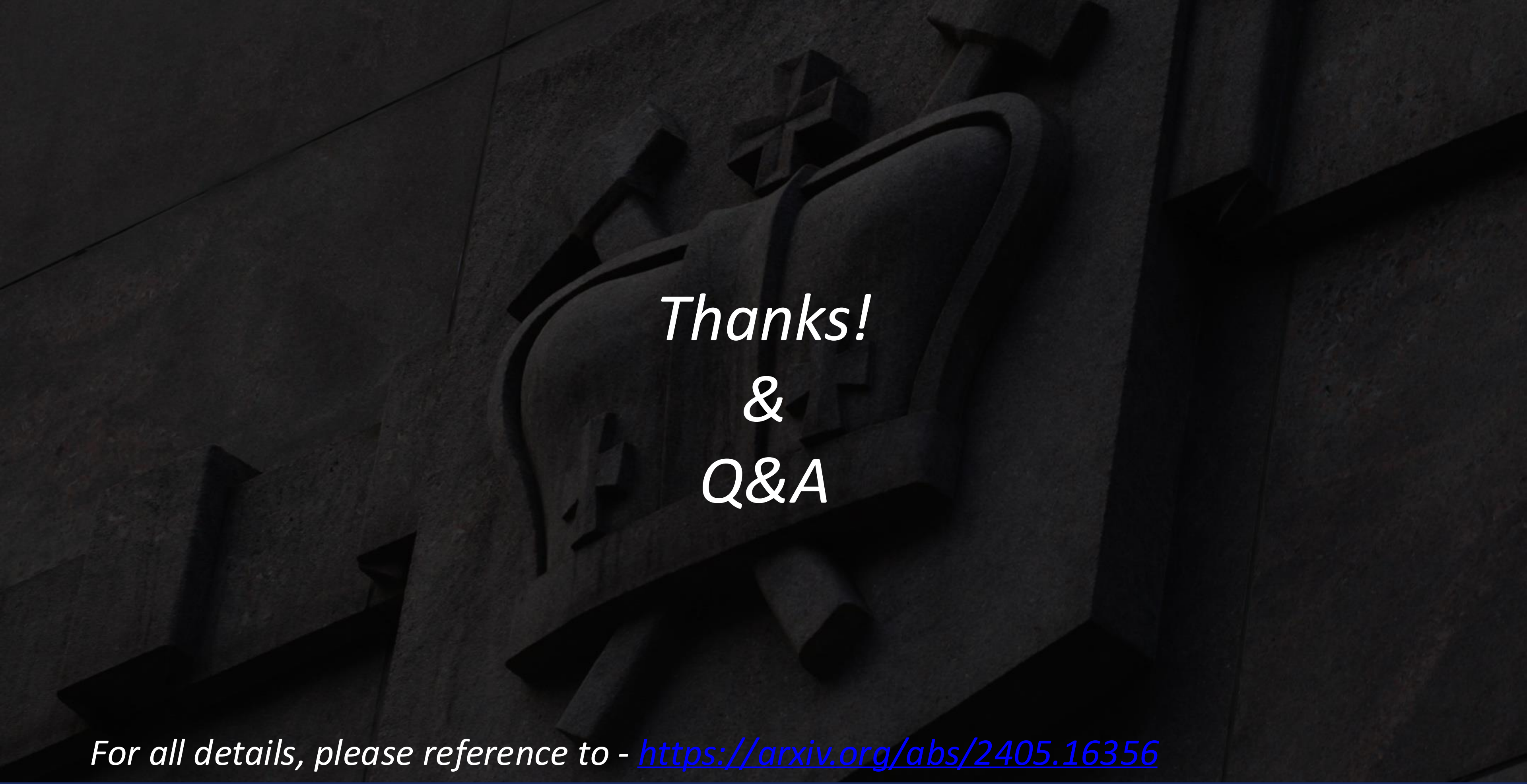

Thanks!

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Q&A

For all details, please reference to - https://arxiv.org/abs/2405.16356