

The Limitations of Proxy Bus Models in Coordinated Transaction Scheduling

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Abstract

Interface scheduling between regional electricity markets can be coordinated through a market mechanism known as coordinated transaction scheduling using a simplified proxy-bus model. This note paper closes a theoretical gap by analyzing the effectiveness of this proxy-bus model from a foundational market economics perspective. We show that no proxy-bus construction can guarantee that market clearing always moves power from lower-cost regions to higher-cost regions so as to reduce the regional price spread. As a result, the proxy-bus construct can inherently lead to efficiency losses. We validate our theoretical results through numerical simulations. Finally, we discuss potential improvements, such as the use of financial interregional transmission products or incorporating boundary network information and allowing node-to-node bidding to overcome the proxy bus model limitations.

CCS Concepts

• **Hardware** → **Interconnect power issues**; • **Social and professional topics** → *Governmental regulations*; *System management*.

Keywords

Proxy bus, regional market coordination, coordinated transaction scheduling, power market efficiency

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1 Introduction

United States regional electricity markets are operated independently by individual Independent System Operators (ISOs) or Regional Transmission Organizations (RTOs). While the neighboring markets operate separately, their underlying electric grids are interconnected. Therefore, market-to-market coordination mechanism such as the Coordinated Transaction Scheduling (CTS) is needed to manage power exchanges across regional interfaces, with the primary economic objective of scheduling interface flows from a lower-marginal-cost region to a higher-marginal-cost region. In electricity markets, marginal costs are reflected through market

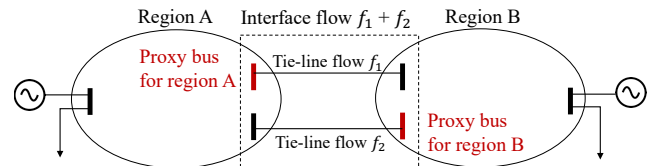


Figure 1: Regional interconnected power network and proxy-bus representation

clearing prices. Accordingly, the CTS objective can be interpreted as reducing the electricity price spread between regions, ideally driving the spread to zero or until the interface limit is reached. However, US ISO/RTO markets typically adopt nodal pricing, which raises a fundamental question: *what constitutes a regional price that can economically drive cross-region power flows?*

To obtain a regional price, the proxy bus concept is introduced, leading to a single proxy bus representation of a region in interregional coordination as illustrated in Fig. 1, whose price is treated as the regional electricity price. For example, at the New England (NE)–New York (NY) interface, Roseton and Sandy Pond serve as the designated proxy buses for NY and NE, respectively [4]. This abstraction originates from the joint optimization of two regional markets, where interregional exchanges may be scheduled by a fully centralized optimizer. In that setting, the proxy bus prices arise endogenously from the joint optimization problem of two regions and thus lead to full economic efficiency.

Under the CTS framework, the proxy-bus model was used to enable CTS bidding and limited information sharing. CTS clearing is therefore conducted using the marginal prices at the designated proxy buses, which must function as static bidding and settlement points. Unlike joint optimization of both regions, proxy bus prices under CTS must guide decentralized decisions under uncertainty and congestion. In practice, transmission networks are far more complex than the proxy bus abstraction, with multiple buses in each region and multiple tie-lines across the interface. Interface flow is the algebraic sum of all tie-line flows, and loop-flow effects may cause individual tie-lines to carry power in directions opposite to the net interface flow. As a result, no single bus price can fully represent a region's economic condition or guarantee that CTS-cleared flows always reduce the regional price spread. This limitation does not arise in joint optimization of two regions, but becomes fundamental under the CTS setting. Consistent with this insight, the ISO-NE Internal Market Monitor reports that nearly 40% of CTS-scheduled flows occur in the “wrong” direction when evaluated against settlement-stage proxy bus prices [2]. This empirical evidence further motivates our research interest in improving



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proxy bus construction to enhance the economic efficiency of CTS clearing.

CTS efficiency has been analyzed using game-theoretic models that benchmark CTS outcomes against centralized tie optimization [3]. Building on this foundation, generalized CTS mechanisms have been proposed that explicitly incorporate full network structure to improve efficiency by minimizing interface trading costs [1]. These studies evaluate the CTS performance under a given proxy bus structure or improve efficiency by altering the market design, but they do not address how the proxy bus itself should be constructed or whether such a construction can be economically consistent. In contrast, we focus on analyzing the effectiveness and fundamental limitations of proxy bus construction theoretically.

In practice, CTS is implemented in two stages to account for uncertainty and imperfect forecasts of demand, renewable generation, and system contingencies: a scheduling stage, conducted prior to the real-time delivery, and a settlement stage, based on market clearing prices at real-time delivery. The scheduling stage schedules interface flows with directions based on forecasted conditions, while the settlement stage updates prices once uncertainty is realized. From this perspective, constructing a proxy bus is equivalent to defining a static mapping from nodal information—such as demand, generation, or locational marginal prices (LMPs)—to a proxy bus price. Such a mapping must be independent of system conditions, including network topology and congestion prices, to ensure that the proxy bus definition remains static between the clearing and settlement stages. The reason is that the proxy bus represents a fixed "virtual location" where market participants place bids. Participants form expectations about future prices based on how the proxy bus is defined; if the mapping were dynamic, those expectations would no longer be well-defined, undermining the proxy bus concept. In practice, topology changes are infrequent and can be treated as fixed within a market interval, whereas congestion prices vary continuously. Therefore, a valid static proxy bus mapping must be independent of the congestion pattern.

Under this setting, our contributions are as follows:

- We close the theoretical gap starting from the market economic foundations to prove that under realistic network conditions, no proxy bus exists within the two-point framework whose price can always drive CTS-cleared power flows from the lower-price region to the higher-price region.
- We further analyze that, within the proxy-bus framework, the best achievable proxy bus construction is necessarily statistically optimal with a quantifiable error. Eliminating this error requires a fundamental shift in market design, specifically, abandoning the proxy-bus model and explicitly incorporating interface network structure. This generalized framework enables node-level bidding and requires coordinated clearing with partial network information shared across regions, which can be analyzed using dual decomposition methods.

The remainder of the paper is structured as follows. Section II presents the model and preliminaries. Section III develops our main theoretical results on proxy bus formation. Section IV reports simulation results, and Section V concludes with practical implications.

2 Model and Formulation

To analyze proxy bus construction, we start from the fundamental joint economic dispatch perspective, which serves as a benchmark for market efficiency. We assume perfect forecasts between the clearing and settlement stages so that the analysis isolates market structure from the effects of forecasting error. We denote transmission lines by the index $l \in L$ and buses by $i, j \in N$. The system is partitioned into two regions, A and B, with node sets denoted by N_A and N_B . Correspondingly, L_A , L_B , and L_{int} denote the sets of transmission lines within region A, within region B, and across the interface, respectively.

At the nodal level, the decision variables include generation g_i , demand d_i , net power injection p_i , and the locational marginal price (LMP) π_i , satisfying the nodal balance condition $g_i - d_i = p_i$. Line-level variables include the power flow f_l and the congestion prices σ_l^- and σ_l^+ associated with the lower and upper flow limits. At the interface level, we define the total interface flow as $f_{\text{int}} = \sum_{l \in L_{\text{int}}} f_l$, the regional marginal prices λ_A and λ_B for regions A and B, respectively, and the interface congestion prices μ^- and μ^+ for the two flow directions.

For notational convenience, we use capital letters to denote vectors in matrix form, including the power flow vector $F \in \mathbb{R}^{|L|}$ and the power injection vector $P \in \mathbb{R}^{|N|}$. To relate nodal injections to line flows, we employ the power transfer distribution factor (PTDF) matrix $H \in \mathbb{R}^{|L| \times |N|}$, which characterizes the sensitivity of each line flow to nodal injection changes [5]. This relationship is given by $F = HP$. Assuming no network losses, the joint economic dispatch problem minimizes total generation cost while satisfying operational constraints, and can be formulated as

$$\min_{g, p} \sum_{i \in N} C_i(g_i) \quad (1a)$$

$$\text{s.t. } g_i - d_i = p_i, \quad \forall i \in N : (\pi_i \in \mathbb{R}), \quad (1b)$$

$$-f_l^{\text{max}} \leq f_l = (HP)_l \leq f_l^{\text{max}}, \quad \forall l \in L : (\sigma_l^-, \sigma_l^+ \in \mathbb{R}), \quad (1c)$$

$$-f_{\text{int}}^{\text{max}} \leq f_{\text{int}} = \sum_{l \in L_{\text{int}}} f_l \leq f_{\text{int}}^{\text{max}} : (\mu^-, \mu^+ \in \mathbb{R}), \quad (1d)$$

$$\sum_{i \in N_A} p_i - f_{\text{int}} = 0 : (\lambda_A \in \mathbb{R}), \quad (1e)$$

$$\sum_{i \in N_B} p_i + f_{\text{int}} = 0 : (\lambda_B \in \mathbb{R}), \quad (1f)$$

$$0 \leq g_i \leq g_i^{\text{max}}, \quad \forall i \in N. \quad (1g)$$

Here, g_i^{max} denotes the generation capacity limit at node i , while f_l^{max} and $f_{\text{int}}^{\text{max}}$ represent the capacity limits of each transmission line and the interface, respectively. In practice, the interface capacity limit is typically smaller than the aggregate capacity of the individual tie-lines, i.e., $\sum_{l \in L_{\text{int}}} f_l^{\text{max}} > f_{\text{int}}^{\text{max}}$. For convenience, we define the net congestion prices as $\mu = \mu^+ - \mu^-$ for the interface and $\sigma_l = \sigma_l^+ - \sigma_l^-$ for line l .

From economic theory, the dual variables associated with the constraints represent shadow prices. Note that constraints (1e) and (1f) jointly enforce system-wide power balance, but are expressed here at the regional level. Their corresponding dual variables, λ_A and λ_B , are therefore interpreted as the regional marginal prices. We

omit explicit discussion of the dual variables associated with the generation capacity constraints (1g), as the marginal value of relaxing generation limits is not central to the proxy bus analysis. This formulation provides the benchmark against which we evaluate the feasibility and limitations of static proxy bus constructions.

3 Main Results

We begin with a benchmark result that characterizes the ideal proxy price from an economic dispatch perspective.

PROPOSITION 3.1. *Ideal proxy price. Under joint economic dispatch, clearing the interface flow based on the regional marginal prices λ_A and λ_B —treated as proxy prices in a two-point network model—always reduces the regional price spread until it reaches zero or the interface capacity constraint becomes binding.*

Proposition 3.1 establishes the benchmark proxy prices whose associated interface clearing is always economically efficient. This result follows directly from the first-order optimality conditions of the joint dispatch problem. From the stationarity conditions with respect to the power injection variables p_i , we obtain

$$0 = \pi_i - \lambda_A + (H^T \sigma)_i, \quad \forall i \in N_A, \quad (2a)$$

$$0 = \pi_i - \lambda_B + (H^T \sigma)_i, \quad \forall i \in N_B. \quad (2b)$$

These conditions show that the regional marginal prices λ_A and λ_B correspond to the *energy components* of locational marginal prices within each region. All nodal LMP differences within a region arise from congestion components captured by the PTDF matrix and line congestion prices. In the absence of congestion, LMPs are uniform within each region. Since network losses are neglected, this decomposition differs slightly from formulations that include marginal loss components.

Economically, λ_A and λ_B can be interpreted as the marginal cost of increasing net export from region A or net import into region B by one unit, respectively. To accommodate such an incremental interchange, the system optimally redispatches generation and demand across buses within one region, subject to the region's internal transmission constraints and interface limits.

While Proposition 3.1 identifies the correct proxy prices from a system-wide optimization perspective, these prices are not directly observable in practice. The central question is therefore whether λ_A and λ_B can be recovered from nodal information through static mappings, consistent with the two-point proxy bus framework used in CTS.

THEOREM 3.2 (IMPOSSIBILITY OF STATIC PROXY BUSES). *Suppose that (i) at least one internal line constraint is binding, (ii) the interface constraint is binding, and (iii) the interface admits loop flows, i.e., multiple tie-lines allow power to be rerouted without changing regional net injections. Fix $\lambda_A = \pi_s$ for a slack bus $s \in N_A$. Then there does not exist a static function*

$$\lambda_B = f_B(\{\pi_i\}_{i \in N_B})$$

that recover the regional marginal prices from nodal LMPs for all operating conditions.

Fixing λ_A as the slack-bus LMP π_s immediately defines a static proxy mapping for region A and serves only to normalize the overall price level. This is without loss of generality, since adding a common

constant to (λ_A, λ_B) does not affect any dispatch decision or price spread. In Theorem 3.2, we focus on LMPs as the nodal information set. This is without loss of generality: if a static mapping fails to exist even when full LMP information is available, it cannot exist under a more restricted information set, e.g., including generation g and demand d . Moreover, if a proxy price depends on line conditions, then those conditions necessarily enter through congestion prices, regardless of what nodal information is considered.

We set the $\lambda_A = \pi_s$ from a market-foundations perspective, where a proxy price is meaningful only if it is consistent with the optimality system: the reported prices must support the optimal dispatch by reflecting marginal system costs and binding network constraints. Equivalently, proxy prices must satisfy the KKT-based coupling relations between energy prices, congestion multipliers, and interface shadow values. Without this coupling, any function of nodal information would merely be a numerical label and would not represent an economic direction in the market.

The economic obstruction behind Theorem 3.2 is the presence of interface loop flow degrees of freedom. When multiple tie-lines connect two regions, power transfers can be routed through different interface paths while preserving the same net injections in each region. Under assumptions (i) and (ii), both internal congestion and interface congestion are active, so congestion rents must be allocated across internal lines, interface lines, and the aggregate interface limit. Assumption (iii) implies that this allocation is not uniquely determined from within-region LMPs: different congestion allocations (across interface and internal constraints) can be compatible with the same regional nodal LMP vector π_B . Consequently, the interface shadow value that drives regional price separation—and therefore the implied regional marginal price λ_B under the slack-side normalization—cannot be expressed as a static function of π_B alone.

Finally, note that if congestion prices on all binding constraints were known, (λ_A, λ_B) could be recovered by the nodal price decomposition (LMP equals an energy component plus congestion components). However, such recovery is necessarily condition-dependent: it varies with which constraints bind and how congestion is distributed across interface paths. This is precisely what violates the static proxy-bus assumption in two-point CTS models, where proxy buses are intended to represent fixed virtual bidding locations where participants place bids and form price expectations.

4 Numerical Study

In this section, we illustrate Theorem 3.2 using the four-bus system shown in Fig. 2. Buses 1–2 belong to region A and buses 3–4 belong to region B. Lines f_1 and f_2 are internal to regions A and B, respectively, while f_3 and f_4 are tie-lines connecting (1, 3) and (2, 4). The aggregate interface flow is defined as $f_{\text{int}} := f_3 + f_4$, with signs following the reference directions in Fig. 2.

We set line limits $f_1^{\max} = 10$, $f_2^{\max} = 6$, $f_3^{\max} = 10$, $f_4^{\max} = 10$, and $f_{\text{int}}^{\max} = 8$. Generation costs are linear: $C_1(g_1) = 4g_1$, $C_3(g_3) = g_3$, and $C_4(g_4) = 8g_4$. Bus 1 is chosen as the slack bus, yielding the normalization $\lambda_A = \pi_1$. Note that this four-bus network is the smallest testbed that satisfies the loop-flow condition in Theorem 3.2.

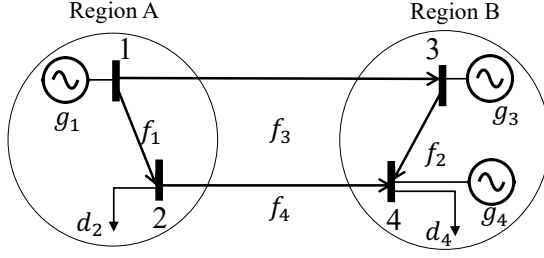


Figure 2: Four buses test system

Table 1: Power flow, LMPs, and regional price spread with different demand scenarios.

Scenario	f_1	f_2	f_3	f_4	π_1	π_2	π_3	π_4	f_{int}	μ
S1	-10	2	-2	10	4	1	8	10	8	2
S2	-2	6	0	8	4	4.7	8	10	8	4.7
S3	-9	1	-1	9	1	1	8	8	8	7
S4	-10	-2	-2	6	4	1	3	2	4	0
S5	-10	6	-	10	4	1	8	10	10	9

Notes: S1: $d_1 = 20, d_2 = 0, d_3 = 24$; S2: $d_1 = 20, d_2 = 50, d_3 = 24$; S3: $d_1 = 10, d_2 = 0, d_3 = 10$; S4: $d_1 = 20, d_2 = 20, d_3 = 4$; S5: $d_1 = 20, d_2 = 20, d_3 = 20$, with tie-line (1, 3) removed. '-' indicates that the corresponding line is not present in the network.

With only three buses, interface flows have a unique routing, leaving no degree of freedom to redistribute flow among tie-lines while preserving regional net injections.

We vary demand levels and, in one case, the network topology to construct five scenarios. For each scenario, Table 1 reports the line flows (f_1, f_2, f_3, f_4), aggregate interface flow f_{int} , nodal LMPs ($\pi_1, \pi_2, \pi_3, \pi_4$), and the interface shadow price (regional price spread) μ .

Scenarios S1–S2 satisfy all three conditions of Theorem 3.2: internal congestion is present, the interface constraint binds ($f_{\text{int}} = f_{\text{int}}^{\text{max}} = 8$), and the two tie-lines create a loop-flow degree of freedom. Across S1–S2, the region B LMP vector is identical, $\pi_B = (\pi_3, \pi_4) = (8, 10)$, and the normalization fixes $\lambda_A = \pi_1 = 4$. If a static mapping $\lambda_B = f_B(\pi_B)$ existed, λ_B would be the same in both scenarios. However, Table 1 shows that the interface shadow price differs ($\mu = 2$ in S1 versus $\mu = 4.7$ in S2), even though π_B and f_{int} remain unchanged. Thus, λ_B cannot be determined from π_B alone, illustrating the impossibility result in Theorem 3.2.

Scenarios S3–S5 illustrate that violating any of the theorem's conditions restores a trivial proxy interpretation. (*No internal congestion: violating (i).*) In S3, the interface is binding, but internal congestion is absent (no internal line limit binds). Consequently, nodal prices are uniform within each region: $\pi_1 = \pi_2$ and $\pi_3 = \pi_4$, so one may take $\lambda_A = \pi_1$ and $\lambda_B = \pi_3$ without ambiguity. (*Interface not binding: violating (ii).*) In S4, the interface is not binding ($f_{\text{int}} = 4 < f_{\text{int}}^{\text{max}}$), so the interface shadow price is $\mu = 0$ and there is no interface-induced price spread. In this case, the regions can be represented by a common energy price, and proxy pricing becomes trivial. (*No loop-flow degree of freedom: violating (iii).*) In S5,

we remove line (1, 3), leaving a single tie-line (2, 4) to carry the interface transfer. The interface then has no alternative routing, so the interface congestion is uniquely associated with that tie-line. A consistent proxy interpretation becomes possible again, e.g., using $\lambda_A = \pi_2$ and $\lambda_B = \pi_3$, with the spread matching the interface shadow value.

5 Conclusion and Discussion

This note paper investigates the economic efficiency of CTS under the prevailing two-point proxy bus framework. By grounding the analysis in the joint economic dispatch foundation, we show that while ideal regional proxy prices exist at the system level, such prices generally cannot be recovered from nodal information through static proxy bus constructions under realistic network conditions. In particular, when loop power flow and binding transmission constraints are present, no proxy bus price can consistently drive CTS-cleared interface flows from the lower-price region to the higher-price region. This result establishes a fundamental limitation of the current CTS design, rather than a deficiency of any specific proxy bus selection rule.

The structural inefficiency induced by the two-point CTS framework motivates the development of dedicated financial instruments, analogous to financial transmission rights (FTRs) in energy markets. These CTS-specific products would be acquired ahead of real-time operation and settle ex post on realized settlement-stage interregional price spreads or proxy-induced congestion rents, without affecting physical dispatch. Unlike CTS bids, which are physical scheduling bids that determine interface flows based on proxy price spreads, such financial instruments would purely redistribute the efficiency losses created by proxy bus structure, improving risk allocation and economic transparency without altering system operations.

If the two-point proxy bus structure is retained, proxy bus design therefore reduces to a statistical approximation problem: proxy prices can be constructed using settlement-stage data to minimize expected misalignment with the true regional marginal price, but an approximation error is fundamentally unavoidable. Eliminating this error requires moving beyond the two-point abstraction and modifying the physical market interface, for example by incorporating boundary network structure, enabling node-to-node transactions across regions, and requiring limited interregional coordination. Unlike financial instruments, which mitigate the consequences of proxy bus simplification, these structural approaches address its root cause by allowing interface flows to be scheduled and priced consistently with the underlying network topology.

References

- [1] Ye Guo, Yuting Ji, and Lang Tong. 2018. Generalized coordinated transaction scheduling: A market approach to seamless interfaces. *IEEE Transactions on Power Systems* 33, 5 (2018), 4683–4693.
- [2] Internal Market Monitor. 2023. *2022 Annual Markets Report*. Technical Report. ISO New England Inc.
- [3] Mariola Ndrio, Subhonmesh Bose, Lang Tong, and Ye Guo. 2022. Coordinated transaction scheduling in multi-area electricity markets: Equilibrium and learning. *IEEE Transactions on Power Systems* 38, 2 (2022), 996–1008.
- [4] Matthew White and Robert Pike. 2011. *Inter-regional Interchange Scheduling: Analysis and Options*. Technical Report. ISO White Paper.
- [5] Allen J Wood, Bruce F Wollenberg, and Gerald B Sheblé. 2013. *Power generation, operation, and control*. John Wiley & sons.

Appendix

A Proof of Proposition 3.1

PROOF. We prove the result by examining the Karush–Kuhn–Tucker (KKT) conditions of the joint economic dispatch problem (1), focusing on the relationship between the regional marginal prices and the interface power flow.

We begin by writing the Lagrangian associated with (1):

$$\begin{aligned} \mathcal{L} = & \sum_{i \in N} C_i(g_i) + \sum_{i \in N} \pi_i(p_i - g_i + d_i) + O(g_i) \\ & + \sum_{l \in L} \sigma_l^- (-f_l - f_l^{\max}) + \sum_{l \in L} \sigma_l^+ (f_l - f_l^{\max}) \\ & + \lambda_A \left(- \sum_{i \in N_A} p_i + f_{\text{int}} \right) + \lambda_B \left(- \sum_{i \in N_B} p_i - f_{\text{int}} \right) \\ & + \mu^- (-f_{\text{int}} - f_{\text{int}}^{\max}) + \mu^+ (f_{\text{int}} - f_{\text{int}}^{\max}), \end{aligned} \quad (3)$$

where $O(g_i)$ collects the terms associated with the generation capacity constraints.

Taking the stationarity condition with respect to the interface flow f_{int} yields

$$\lambda_A - \lambda_B - \mu^- + \mu^+ = 0. \quad (4)$$

This condition links the regional marginal price difference to the shadow prices of the interface capacity constraint. We now examine the implications of (4) under complementary slackness. From the complementary slackness conditions associated with constraint (1d), the following cases arise:

- **Uncongested interface:** If $|f_{\text{int}}| < f_{\text{int}}^{\max}$, then $\mu^- = \mu^+ = 0$. Equation (4) implies $\lambda_A = \lambda_B$, indicating that the interface flow equalizes regional marginal prices.
- **Positive-direction congestion:** If $f_{\text{int}} = f_{\text{int}}^{\max}$, then $\mu^+ > 0$ and $\mu^- = 0$. Equation (4) implies $\lambda_A < \lambda_B$, meaning power flows from region A (lower price) to region B (higher price) until the interface capacity is binding.
- **Negative-direction congestion:** If $f_{\text{int}} = -f_{\text{int}}^{\max}$, then $\mu^- > 0$ and $\mu^+ = 0$. Equation (4) implies $\lambda_A > \lambda_B$, meaning power flows from region B to region A until the interface capacity is binding.

In all cases, the sign of the interface power flow is consistent with reducing the regional marginal price difference $\lambda_B - \lambda_A$. Specifically, the interface flow either eliminates the price spread entirely or saturates at the interface capacity limit. This establishes that λ_A and λ_B serve as ideal proxy prices in a two-point CTS framework, completing the proof. $\square$

B Proof of Theorem 3.2

PROOF. We prove the theorem in three steps. Step 1 shows that a static mapping can be constructed for the region containing the slack bus (without using congestion prices). Step 2 shows that, under the stated conditions, no static mapping exists for the other region when the mapping is required to be independent of line congestion conditions. Step 3 characterizes three degenerate cases in which static mappings do exist, thereby clarifying the role of each theorem condition.

Notation. Let $H \in \mathbb{R}^{|L| \times |N|}$ be the PTDF matrix under a fixed slack bus. For a region $r \in \{A, B\}$, let H_r be the submatrix formed by restricting columns of H to buses in N_r , and let π_r denote the nodal LMP vector in region r . Recall the stationarity conditions with respect to injections p_i :

$$\pi_r = \lambda_r \mathbf{1} - H_r^T \sigma, \quad r \in \{A, B\}, \quad (5)$$

where $\sigma := \sigma^+ - \sigma^-$ denotes the net line-congestion multiplier.

Normalization and economic meaning of proxy prices. The pair (λ_A, λ_B) arises as dual variables of the *regional balance* equalities introduced to represent an interface schedule. As in DC power flow where one fixes a slack angle to remove a reference ambiguity, regional proxy prices are identifiable only up to a common additive shift: adding a constant κ to both (λ_A, λ_B) does not change the KKT system for primal variables nor the economically meaningful nodal prices π , and leaves the spread $\lambda_B - \lambda_A$ invariant. Therefore, selecting a slack-side convention (e.g., setting λ_A equal to a slack-bus LMP in region A) is without loss of generality and serves only as a normalization that pins down the price level. Moreover, a proxy price is economically meaningful only if it remains consistent with the KKT coupling; otherwise one could assign arbitrary numbers to (λ_A, λ_B) that do not represent marginal costs or incentives.

Step 1: Existence of a static proxy mapping for the slack-bus region. Assume the slack bus s lies in region A. Let $w_A \in \mathbb{R}^{|N_A|}$ satisfy $\mathbf{1}^T w_A = 1$. Multiplying (5) for $r = A$ by w_A yields

$$w_A^T \pi_A = \lambda_A \mathbf{1}^T w_A - w_A^T H_A^T \sigma = \lambda_A - \sigma^T (H_A w_A). \quad (6)$$

Hence, if w_A is chosen such that

$$\mathbf{1}^T w_A = 1, \quad H_A w_A = 0, \quad (7)$$

then (6) implies the *static* representation

$$\lambda_A = f_A(\pi_A) := w_A^T \pi_A, \quad (8)$$

which is independent of line congestion duals σ .

Such a w_A exists because the PTDF column corresponding to the slack bus is zero, i.e., $H_{\cdot s} = 0$, and since $s \in N_A$ we have $H_A e_s = 0$ with $\mathbf{1}^T e_s = 1$. Thus $w_A = e_s$ satisfies (7) and yields the canonical proxy rule $\lambda_A = \pi_s$.

More generally, one may construct w_A by projecting any vector $\tilde{w}$ onto $\ker(H_A)$ and normalizing:

$$\tilde{w} := \text{Proj}_{\ker(H_A)}(\tilde{w}), \quad w_A := \frac{\tilde{w}}{\mathbf{1}^T \tilde{w}}, \quad (9)$$

where $\text{Proj}_{\ker(H_A)}$ denotes the orthogonal projection onto $\ker(H_A)$ (well-defined since $\ker(H_A)$ is a linear subspace). This completes Step 1.

Slack-bus invariance. The physical OPF and its nodal LMPs π are invariant to the choice of slack bus; changing the slack only changes the PTDF representation used to compute flows. Our use of a slack bus $s \in A$ is therefore purely a normalization (reference choice) and does not affect the spread $\lambda_B - \lambda_A$ nor the existence/nonexistence claims below.

Step 2: Nonexistence of a static proxy mapping for the non-slack region under the theorem conditions. We now show that, under assumptions (i)–(iii), once the slack-side proxy rule $\lambda_A = f_A(\pi_A)$ is fixed as in Step 1, there cannot exist a static function $\lambda_B = f_B(\pi_B)$ that is independent of line congestion conditions.

Let $e_{\text{int}} \in \mathbb{R}^{|L|}$ denote the indicator vector of interface tie-lines: $(e_{\text{int}})_\ell = 1$ if $\ell \in L_{\text{int}}$ and $(e_{\text{int}})_\ell = 0$ otherwise, so that the aggregate interface flow is

$$f_{\text{int}} = \sum_{\ell \in L_{\text{int}}} f_\ell = e_{\text{int}}^\top F. \quad (10)$$

Fix any feasible instance and consider any pair (λ_B, π_B) . From the stationarity condition (5) with $r = B$ we have

$$H_B^\top \sigma = \lambda_B \mathbf{1} - \pi_B. \quad (11)$$

Thus the set of line-congestion dual vectors that are compatible with (λ_B, π_B) is the affine space

$$\mathcal{M}(\lambda_B, \pi_B) := \{\sigma : H_B^\top \sigma = \lambda_B \mathbf{1} - \pi_B\} = \sigma_0 + \ker(H_B^\top), \quad (12)$$

where σ_0 is any particular solution.

Assumption (iii) states that the interface admits a loop-flow dual direction invisible to region B:

$$\exists \delta \in \ker(H_B^\top) \text{ such that } e_{\text{int}}^\top \delta \neq 0. \quad (13)$$

Fix any $\sigma_a \in \mathcal{M}(\lambda_B, \pi_B)$ and define $\sigma_b := \sigma_a + \epsilon \delta$ for some scalar ϵ . Since $H_B^\top \delta = 0$, we have $H_B^\top \sigma_b = H_B^\top \sigma_a$, and therefore σ_a and σ_b are both compatible with the same pair (λ_B, π_B) through (11).

We now incorporate the interface KKT coupling. By Assumption (ii), the aggregate interface constraint is binding, i.e., $|f_{\text{int}}| = f_{\text{int}}^{\max}$, and thus complementary slackness allows a nonzero interface multiplier: either $\mu^+ > 0$ (if $f_{\text{int}} = f_{\text{int}}^{\max}$) or $\mu^- > 0$ (if $f_{\text{int}} = -f_{\text{int}}^{\max}$). Stationarity with respect to the interface flow implies the spread relation

$$\lambda_A - \lambda_B = \mu^- - \mu^+. \quad (14)$$

Moreover, Assumption (i) ensures that at least one internal line constraint is binding, so that the congestion multipliers σ are not identically zero and the feasible dual set in (12) is nontrivial.

Because (13) provides a perturbation direction with nonzero interface component $e_{\text{int}}^\top \delta \neq 0$, varying ϵ moves the dual congestion pattern along a loop-flow direction that is *invisible* to region B prices (it leaves $H_B^\top \sigma$ unchanged) but changes the interface component of the dual system that must be balanced by (μ^+, μ^-) . Hence there exist dual-feasible KKT tuples with the same π_B but different interface multipliers (μ^+, μ^-) when the interface is binding.

Finally, Step 1 fixes the normalization by selecting a slack-side proxy rule $\lambda_A = f_A(\pi_A)$ (in particular, $\lambda_A = \pi_s$ for a slack bus $s \in A$), which is independent of the loop-flow dual perturbations above. Therefore, since λ_A is pinned while (μ^+, μ^-) can vary, (??) forces different implied values of λ_B while π_B remains unchanged. This contradicts the existence of a congestion-independent static mapping $\lambda_B = f_B(\pi_B)$.

In summary, (i) guarantees nontrivial congestion dual structure, (ii) activates the interface multiplier channel μ through complementary slackness, and (iii) provides a loop-flow direction that decouples the interface dual adjustments from π_B .

Step 3: Degenerate cases where static mappings exist. We briefly verify that dropping any of the theorem conditions recovers cases in which static mappings can be defined.

(i) *No loop flows in the interface.* The first case that excludes loop flows is when $|L_{\text{int}}| = 1$. In this case, f_{int} equals the unique tie-line flow, and the interface constraint coincides with that line constraint. The corresponding congestion price therefore satisfies $\mu = \sigma_{\text{int}}$, eliminating the loop-flow degrees of freedom responsible for (13). As a result, one can select boundary buses such that $\lambda_A - \lambda_B$ equals the LMP difference at the endpoints of the unique tie-line.

A second case arises in small or radially connected interface networks. For example, in a three-bus system with one slack-region bus connected to two buses in the other region, the interface may consist of multiple tie-lines but still admit no nontrivial loop-flow directions. In such topologies, the distribution of power across tie-lines is uniquely determined by regional net injections. Consequently, the interface flow decomposition is fixed, and a static proxy mapping may exist despite the presence of multiple tie-lines.

(ii) *No internal congestion.* If all line-flow constraints (1c) are slack, then $\sigma = 0$. Equation (5) yields $\pi_i = \lambda_A$ for all $i \in N_A$ and $\pi_i = \lambda_B$ for all $i \in N_B$, so the mappings exist trivially.

(iii) *Unconstrained interface.* If the interface constraint (1d) is slack, then $\mu = 0$ and Proposition 3.1 implies $\lambda_A = \lambda_B$. Hence λ_B can be recovered from λ_A (and vice versa), and the proxy mapping requirement becomes vacuous.

These cases highlight that the obstruction to a static proxy mapping is precisely the loop-flow-induced coupling between interface and internal congestion prices when multiple tie-lines and binding constraints are present. $\square$