Columbia Engineering
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Optimal Offering Strategy of a Price-Taking Virtual Power Plant (VPP)

--2023 EEE symposium

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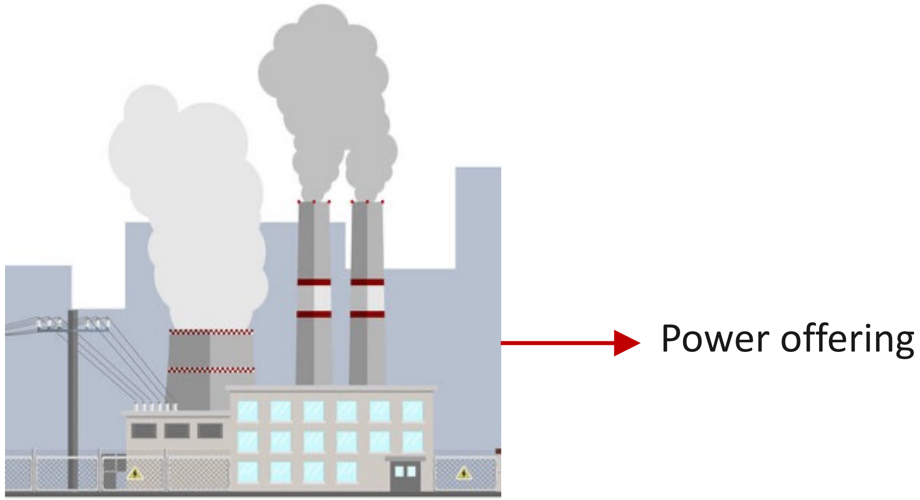
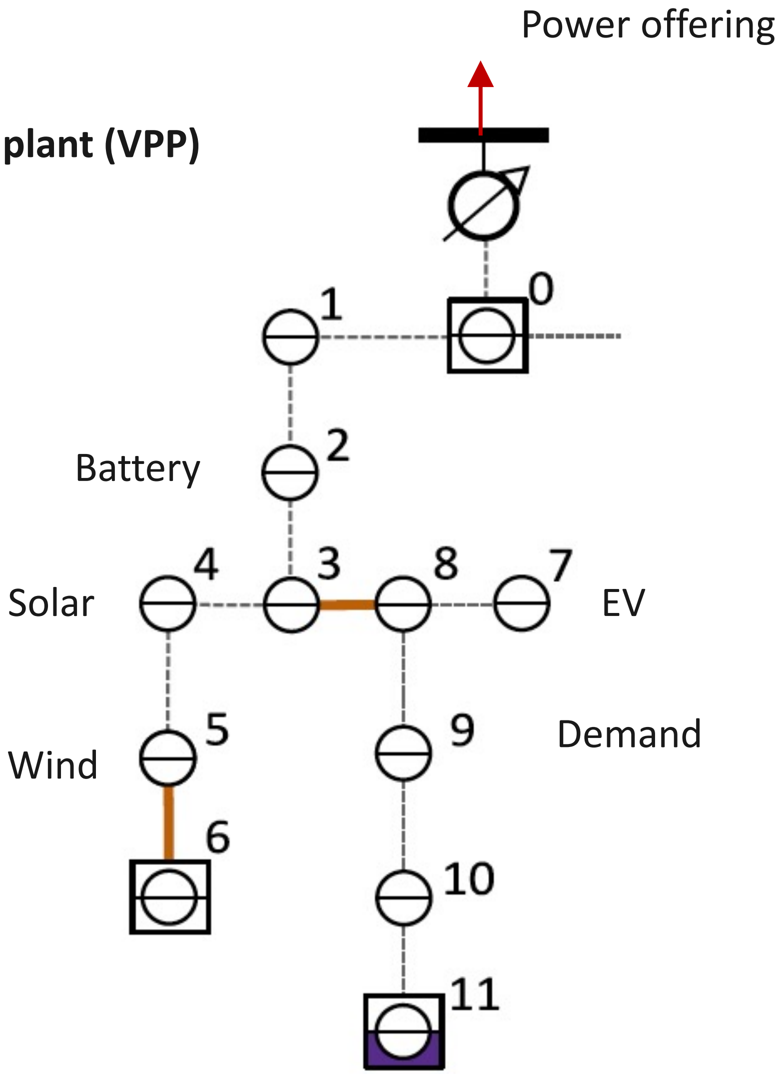
What is Virtual Power Plant

Power plant--Generator



Virtual power plant (VPP)



R. Mieth and Y. Dvorkin, "Distribution Electricity Pricing Under Uncertainty," in IEEE Transactions on Power Systems, vol. 35, no. 3, pp. 2325-2338, May 2020

What is the significant difference between the VPP and the traditional generator?

Content

Background and contribution

Why this topic?

What are the research gaps?

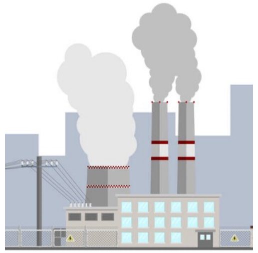
What did we do?

Background

Generator’s profit

Profit = revenue of selling power + cost of generating power

Day-ahead market

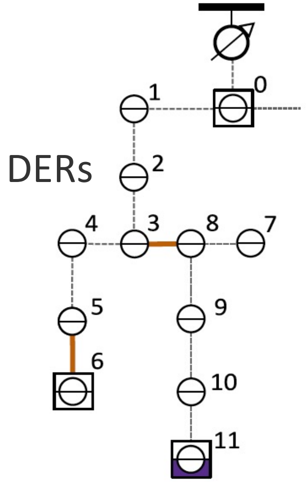


Traditional Generator

Price-quantity pair

Market

Power offering & clearing price

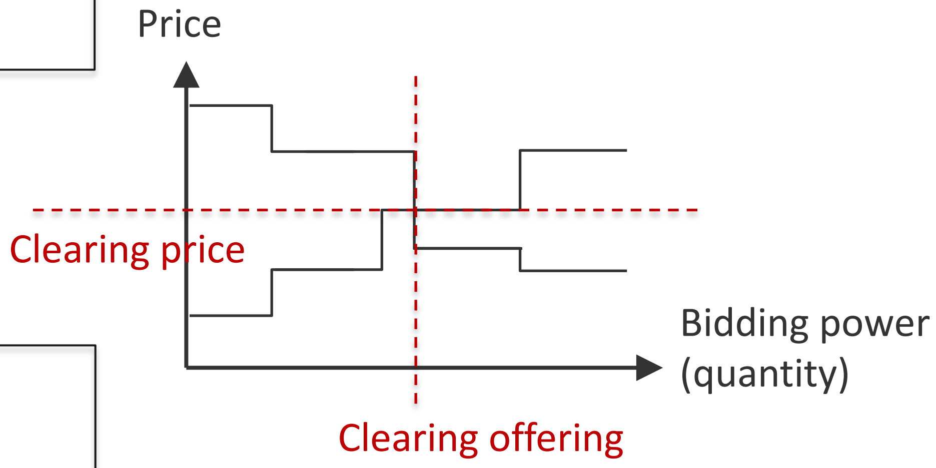


Virtual power plant (including DERs)

Price-quantity pair

Market

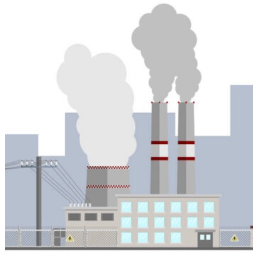
Power offering & clearing price



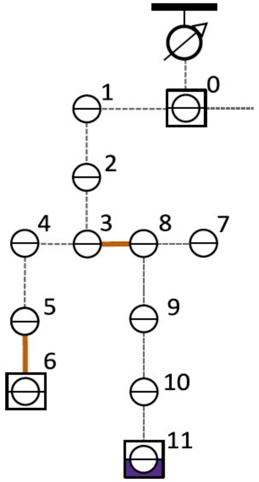
R. Mieth and Y. Dvorkin, "Distribution Electricity Pricing Under Uncertainty," in IEEE Transactions on Power Systems, vol. 35, no. 3, pp. 2325-2338, May 2020

Background

Real-time market



Traditional Generator: Generate required power offering according to the day-ahead market clearing results



Virtual power plant: Change DERs dispatch after price realization in real-time market

- Providing the same offering power
- Minimize generation cost

Cost is determined by fuel cost

Cost is determined by profit maximization considering Real-time market price uncertainty

Gaps & Contribution

Current research

Model price uncertainty under simplified assumptions of stochastic price evolution:
- Uncertainty intervals
- Finite samples of price scenarios

Compromise the optimality of the VPP's power offering

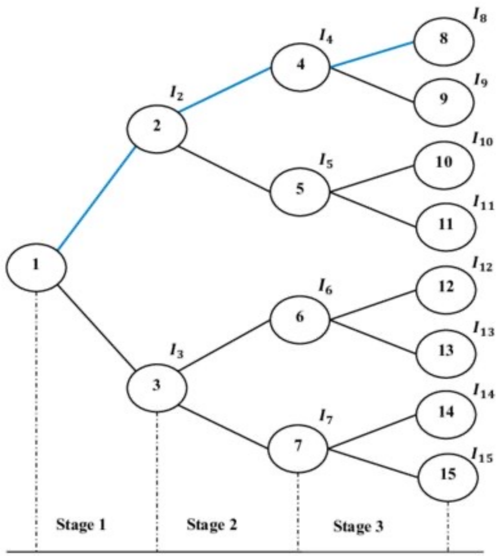
Contribution

Model: VPP offering strategy is determined by profit maximization

$$\max_x \left(f(x) - \min_y g_x(y) \right)$$

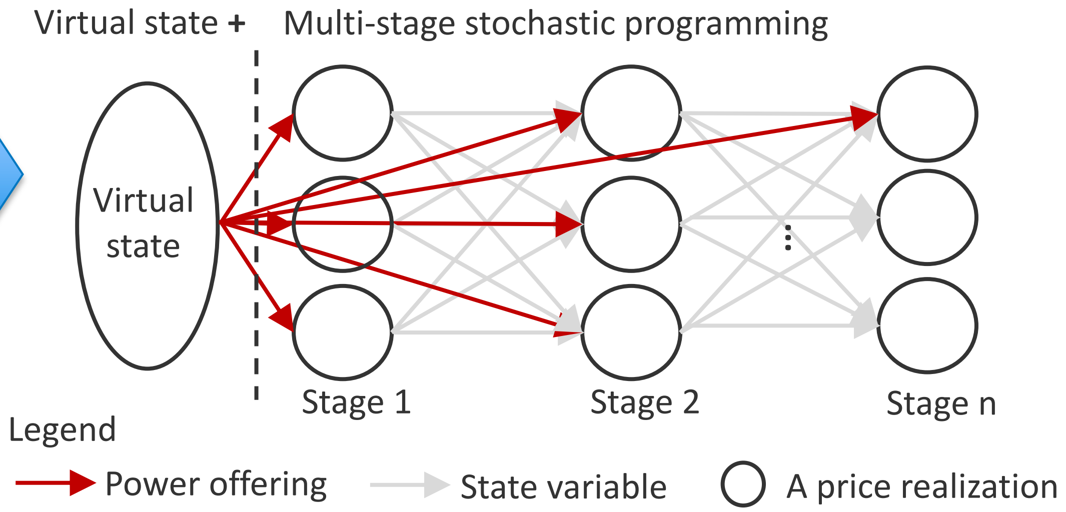
x – power offering
y – DERs control

Technology challenge: exponential price scenario – curse of dimensionality



From Probabilistic Seasonal Streamflow Forecasts to Optimal Reservoir Operations: A Stochastic Programming Approach

Formulation: virtual stage + 24 stages



Formulation

How to formulate the exponential price scenarios?

Formulation

Multi-stage stochastic programming formulation

Simplified

$$\max_x (f(x) - \min_y g_x(y))$$

Revenue   Cost

Full

$$\max_{\boldsymbol{p}} \left\{ \sum_{t=1}^{24} \sum_{s=1}^{|\boldsymbol{\pi}_t|} \mathbb{E}_{\boldsymbol{\pi}_t}[\boldsymbol{\pi}^\top \boldsymbol{p}_t] - \mathbb{E}_{\boldsymbol{\pi}_1} \left[\min_{x_1 \in F_1(p_1)} \left\{ c_1(x_1) - \cdots - \mathbb{E}_{\boldsymbol{p}_{24}|p_{[1,23]}} \left[\min_{x_{24} \in F_{24}(x_{23}, p_{24})} c_{24}(x_{24}) \right] \right\} \right] \right\}$$

Virtual stage: determine the power offering corresponding to prices, that is to get the price-quantity pair, maximize profit

Subproblem:

$$\max_{p_{1-24,s}} f(p_{1-24,s}) + \sum_{m \in \chi_0} q_{0,m} Q_m(p_{1-24,s})$$

$$f = \sum_{t=1}^{24} \sum_{s=1}^{|\boldsymbol{\pi}_t|} \pi_{ts} p_{ts}$$

Cost-to-go function= Transition probability * Optimal value function

Stage 1: determine first time slot DERs dispatch, minimize generation cost

⋮

Stage 24: determine 24 time slots DERs dispatch, minimize generation cost

Subproblem:

$$Q_1(p_{1-24}) = \min_{x_1, y_1} \{C_1(x_1) + \sum_{m \in \chi_1} q_{1,m} Q_m(x_1, p_{1-24})\}$$

$$C = C_{b,s,t} + C_{dg,s,t} + C_{cur,s,t}$$

$$C_{b,s,t} = \lambda^{deg}(\eta^c p^c_{s,t} + p^d_{s,t}/\eta^d)$$ Cost of battery

$$C_{dg,s,t} = a(p^{dg}_{s,t}) + b$$ distributed generator

$$C_{cur,s,t} = a(p^{cur}_{s,t}) + b$$ demand curtailment

S.t.

$$\sum p_n = L_n$$ Power balance

$$p_{dg,min} \le p^{dg}_{s,t} \le p_{dg,max}$$ Generation limit

$$p_{dg,min} - RR \le p^{dg}_{s,t} - p^{dg}_{s,t-1} \le p_{dg,max} + RR$$ Ramp rate

$$SOC_{s,t} - SOC_{s,t-1} = (\eta^c p^c_{s,t} - p^d_{s,t}/\eta^d)$$ Soc transition

$$0 \le p^c_{s,t}, p^d_{s,t} \le p_{b,max}$$ Charge

$$0 \le SOC_{s,t} \le SOC_{max}$$ Soc

$$0 \le p^{cur}_{s,t} \le p_{cur,max}$$ Curtailment

Network constraints

Solution algorithm

How did we reduce the price scenarios?

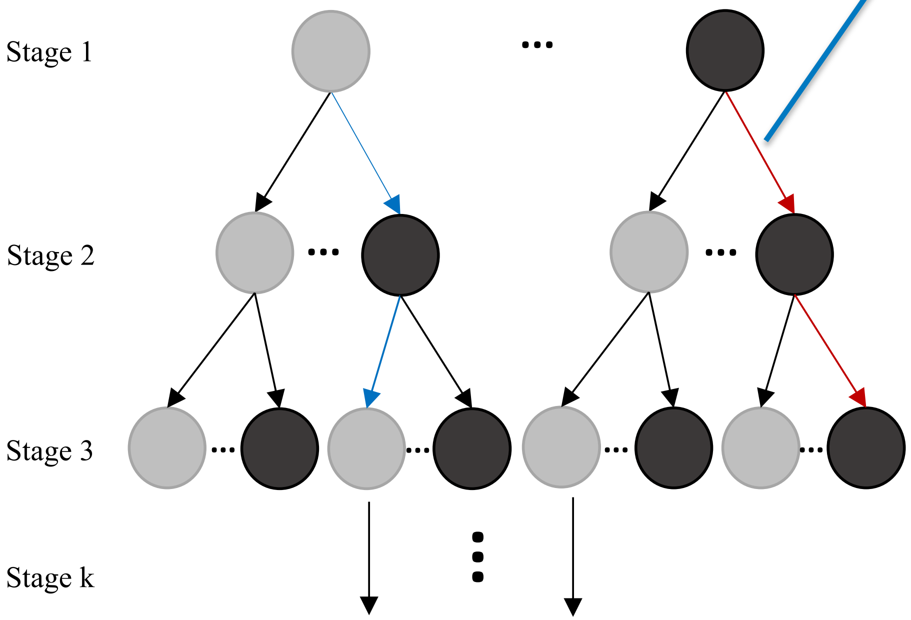
How did we approximate the value function and solve it?

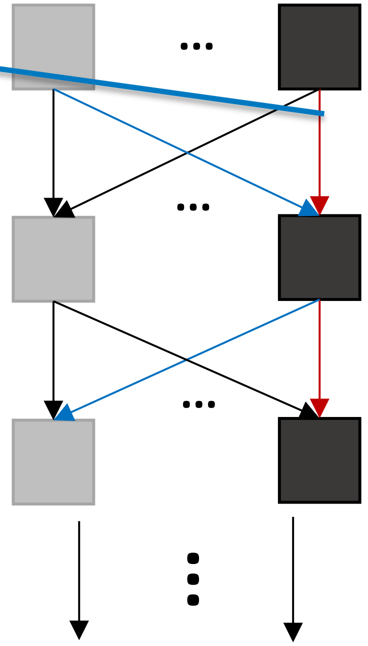
How to deal with the virtual stage

Solution Algorithm

Scenario tree & Policy graph

One line from stages 1-24 indicates a price trajectory, which is exponential.
Corresponding to the transition trajectory in the policy graph



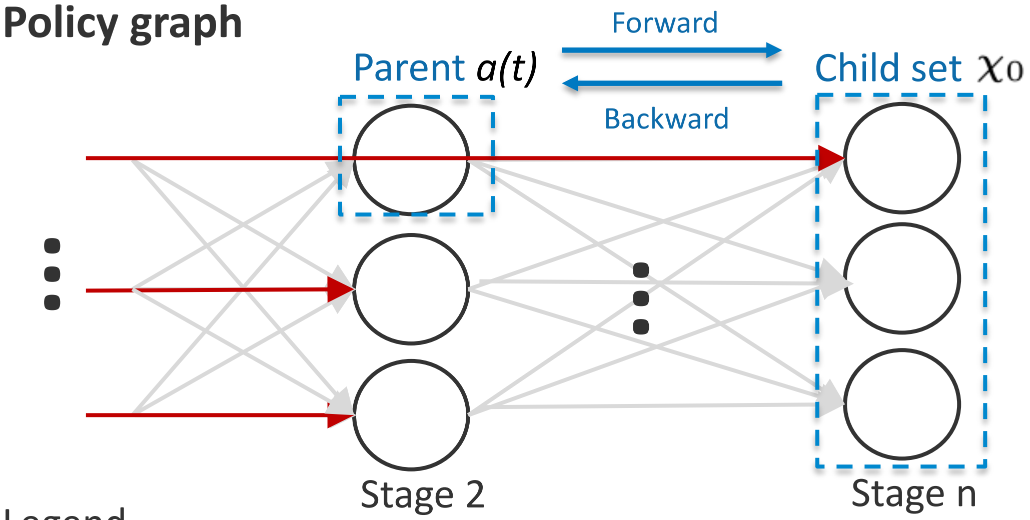


Each node is a price realization, including a decision rule *U=f(x,w)*, give one uncertain realization *w*, and state variable (time coupled variable) *x*, and get a different decision *U*.

Exponential price scenario → **Finite policy graph**

Solution Algorithm

Policy graph



Legend
Power offering — State variable — ○ A price realization

- The first virtual stage determines the power offering corresponding to prices realization
- Transmit power offering to each corresponding price realization
- When VPP changes DERs dispatch, it should follow the power offering determined in the virtual stage

Solution algorithm – stochastic dual dynamic programming (SDDP)

Each iteration *i*:

- Forward: Update state variable x_t following *t=1* to *t=24.* Approximate lower bound of the value function by solving the following optimization model. Passing x_t to child node's (*m*) problem.
- Backward: Solve suitable relaxation to get cuts following *t=24* to *t=1*, update the cost-to-go function by adding cuts from all child nodes to parent nodes, change $\psi_{t,i}$ to $\psi_{t,i+1}$, then solve the suitable relaxation of the updated problem again.

Iteration *i* to *i+1*

Need to know the parent nodes' *a(t)* state variable

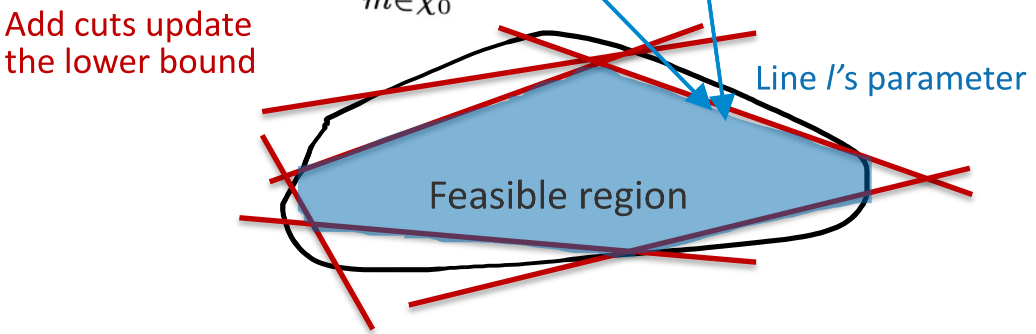
$$\underline{Q}_{t,i}(p_{1-24}, x_{a(t)}, \psi_{t,i}) = \min_{x_t} C_t(x_t, p_{1-24}) + \psi_{t,i}(x_t, p_{1-24})$$

Expected cost-to-go function

$$\text{s.t.}(p_{1-24}, x_t, y_t) \in F_t$$

$$\psi_{t,i}(x_t, p_{1-24}) = \min\{\theta_t : \ \theta_t \geq LB_t,$$

$$\theta_t \geq \sum_{m \in \chi_0} q_{t,m}(v_{m,l} + {\pi_{m,l}}^T x_t), \forall l = 1, 2, i-1\}$$



Solution Algorithm

To summarize

- The forward problem is to solve the optimization model to get the optimal solution: state variable, control variable, and cost-to-go function.
- Update statistical upper bound
- The backward is to solve the relaxation problem to get cuts, and add cuts to update (reduce) the feasible region for the forward optimization. (cuts only cut infeasible area)
- Update lower bound

/* (Statistical) upper bound update */

$\hat{\mu} \leftarrow \frac{1}{M}\sum_{k=1}^{M} u^k$ and $\hat{\sigma}^2 \leftarrow \frac{1}{M-1}\sum_{k=1}^{M}(u^k - \hat{\mu})^2$

$UB \leftarrow \hat{\mu} + z_{\alpha/2}\frac{\hat{\sigma}}{\sqrt{M}}$

/* Lower bound update */

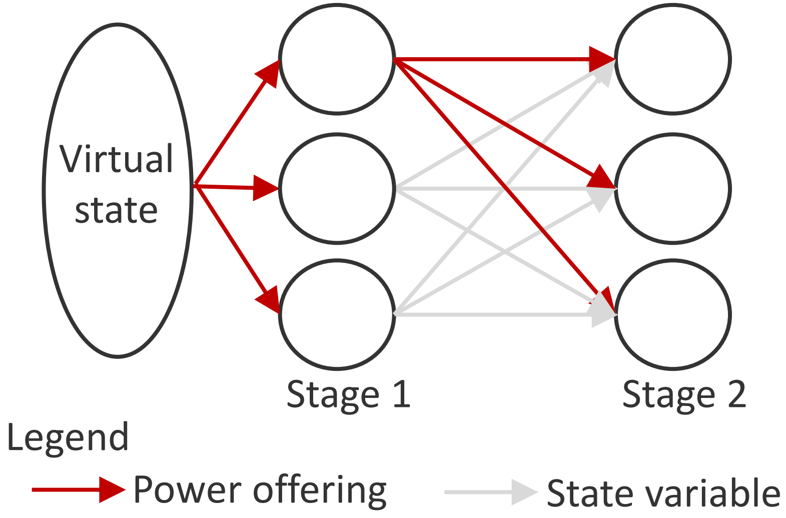
solve $P_1^i(\bar{x}_0, \psi_1^{i+1})$ and set LB to the optimal value

$i \leftarrow i + 1$

Optimal value of forward problem at node 1 is the lower bound

Virtual stage – copying variable

Copying power offering variable determined by the virtual stage and transmitted to following stages.



Keep the power offering as a constant and transmit it to next stage step by step

Zou, J., Ahmed, S., & Sun, X. A. (2019). Stochastic dual dynamic integer programming. Mathematical Programming, 175, 461-502.

Solution Algorithm

Virtual stage – zeroth order stochastic gradient descent

Algorithm 1 Zeroth-order gradient descent algorithm

1: Initialization: perturbation u, number of points for gradient estimation M, initial power offering $\boldsymbol{p}$, convergence criteria ϵ, learning rate h

2: **repeat**

3: Call SDDP function ($f(\boldsymbol{p})$) get optimal DERs dispatch and minimal cost under current offering

4: **for** i=1:M **do**

5: Randomly generate permutation vector $\boldsymbol{z}$, where $\mathbb{E}[\|z\|] = 1$

6: $\boldsymbol{p}^+ = \boldsymbol{p} + u * z$

7: $\boldsymbol{p}^- = \boldsymbol{p} - u * z$

8: $\nabla f_i = (f(\boldsymbol{p}^+) - f(\boldsymbol{p}^-))/2u$

9: **end for**

10: $\nabla f = -\nabla f_{\text{profit}} + \sum_{i \in M} f(i)/M$

11: $\boldsymbol{p} = \boldsymbol{p} - h * \nabla f$

12: Project $\boldsymbol{p}$ into box constraint $[p_{\min}, p_{\max}]$

13: **until** $\|\nabla f\| \le \epsilon$

14: Get price-power offering pairs considering DERs cost

Create an outer loop using zeroth-order stochastic gradient descent to determine the offering power

$$\max_{\boldsymbol{p}} \Big\{ \sum_{t=1}^{24}\sum_{s=1}^{|\boldsymbol{\pi}_t|} \mathbb{E}_{\boldsymbol{\pi}_t}[\boldsymbol{\pi}^\top \boldsymbol{p}_t] - \mathbb{E}_{\pi_1}\Big[\min_{x_1\in F_1(p_1)}\Big\{c_1(x_1) - \cdots - \mathbb{E}_{p_{24}|p_{[1,23]}}\Big[\min_{x_{24}\in F_{24}(x_{23},p_{24})} c_{24}(x_{24})\Big\}\Big]\Big\}\Big\} \quad (1)$$

Cost from SDDP

Zeroth-order method calculates gradient

Update decision variables

Projection dealing with box constraint

Results and future works

What is the performance of our method?

Next step?

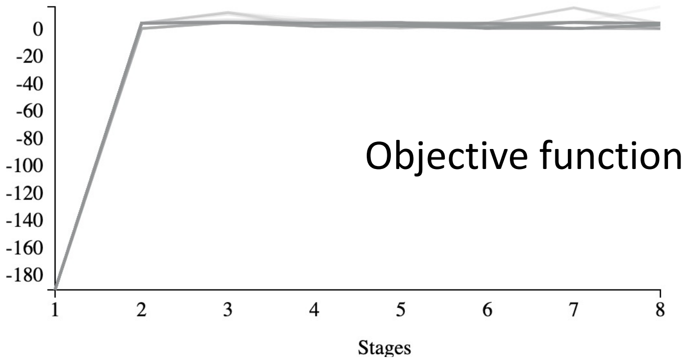
Results

Basic setting

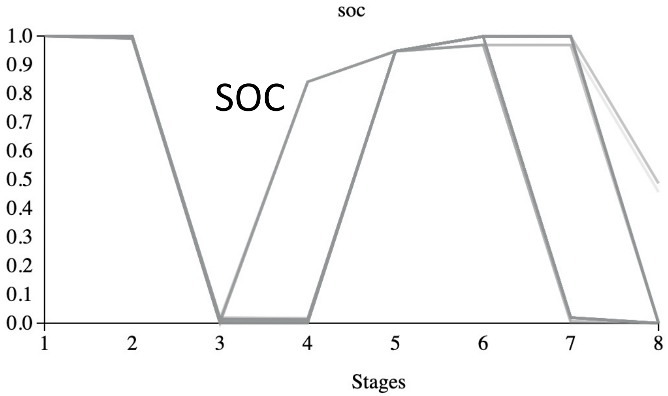
- 1 virtual stage + n stages, each stage includes 5 price scenarios
- 15-bus power network with loads, 6 distributed generators, and 2 batteries
- Using parallel computing and servers to compute

Copying variable - 8 stages (hours)

6 minutes running

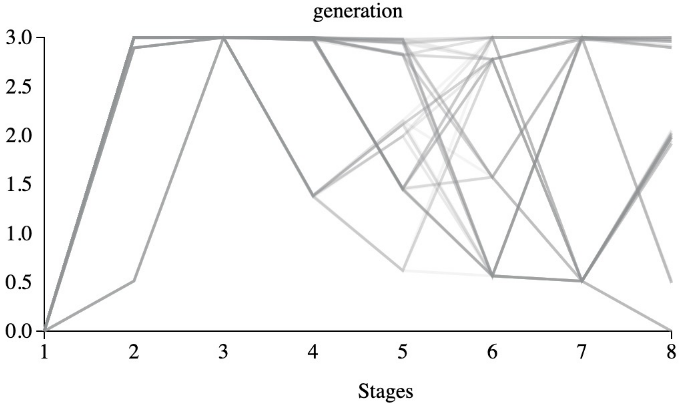


Objective function value



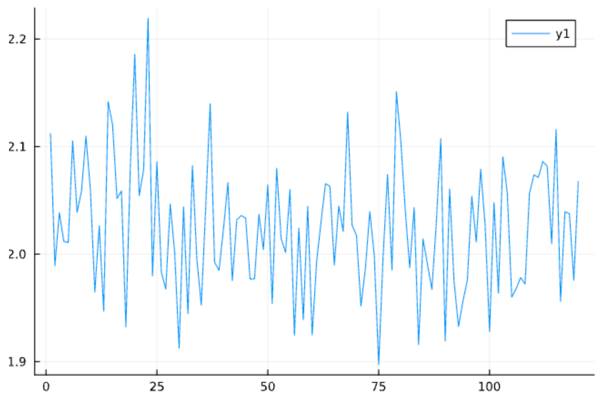
SOC

Distributed generator's generation

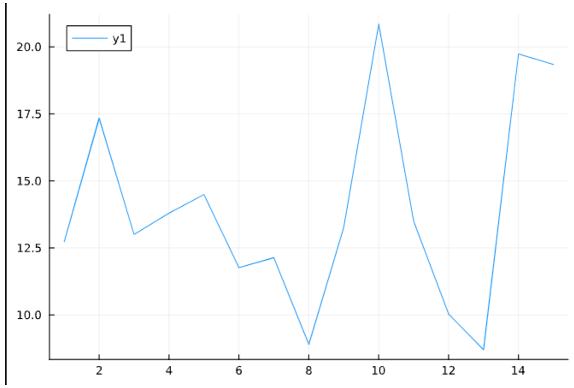


Zeroth-order stochastic gradient descent – 24 hours

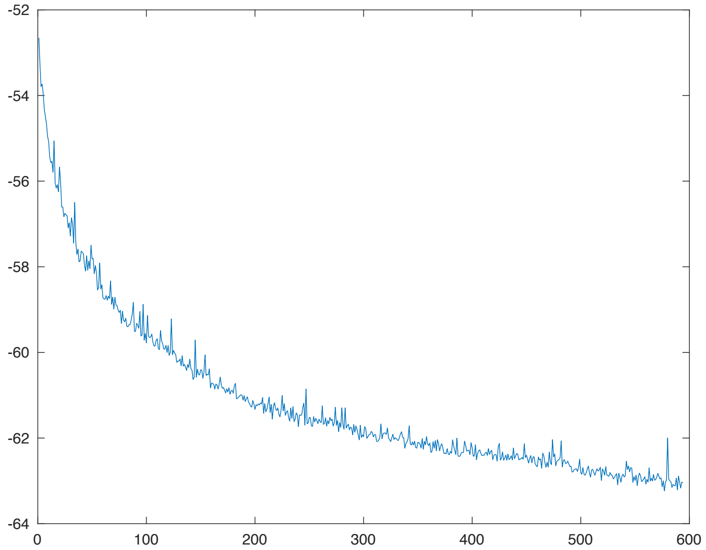
1 hour running



Power Offering



Stochastic gradient recording



Objective function value

Future works

- Update the solution algorithm by adding cuts to the approximate subgradient of the value function, use the proximal bundle method to update the power offering
- Using reinforcement learning to improve the algorithm
- Compare different algorithms' efficiency with regard to this problem
- Including more DERs, and exploring the usage in large power network

Reference

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Go back to the first question

The significant differences between VPP and traditional generators are:

- Stochastic or deterministic generation cost;
- VPP can change DERs dispatch after price realization in the real-time market;
- VPP’s offering strategy should consider future price uncertainty
- Multistage stochastic modeling
- Complicated solution algorithm

Thank you